



## PROBLEM:

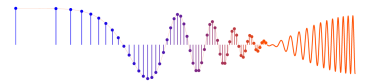
We now have four ways of describing an LTI system: the difference equation with filter coefficients  $\{b_k\}$ ; the impulse response,  $h[n]$ ; the frequency response,  $H(e^{j\hat{\omega}})$ ; and the system function,  $H(z)$ . In the following, you are given one of these representations and you must find the system function  $H(z)$ .

(a)  $y[n] = 3(x[n] - x[n - 3]).$

(b)  $h[n] = -\delta[n] - \delta[n - 1] - \delta[n - 2] - \delta[n - 3].$

(c)  $H(e^{j\hat{\omega}}) = [2j \sin(2\hat{\omega})]e^{-j3\hat{\omega}}.$

(d)  $h[n] = \delta[n] + \delta[n - 3].$



(a)  $y[n] = 3(x[n] - x[n-3])$

$$y[n] = 3x[n] - 3x[n-3]$$

$$h[n] = 3\delta[n] - 3\delta[n-3]$$

$$H(z) = 3 - 3z^{-3}$$

(b)

$$h[n] = -\delta[n] - \delta[n-1] - \delta[n-2] - \delta[n-3]$$

$$H(z) = -1 - z^{-1} - z^{-2} - z^{-3}$$

(c)

$$H(e^{j\hat{\omega}}) = [2j \sin(2\hat{\omega})] e^{-j3\hat{\omega}}$$

We use the fact that  $H(e^{j\hat{\omega}}) = H(z)$ , together with the Euler identity  $\sin \theta = \frac{e^{j\theta} - e^{-j\theta}}{2j}$ .

$$H(e^{j\hat{\omega}}) = 2j e^{-j3\hat{\omega}} \left( \frac{e^{j2\hat{\omega}} - e^{-j2\hat{\omega}}}{2j} \right)$$

$$= e^{-j3\hat{\omega}} (e^{j2\hat{\omega}} - e^{-j2\hat{\omega}})$$

$$= e^{-j\hat{\omega}} - e^{-j5\hat{\omega}}$$

$$H(z) = z^{-1} - z^{-5}$$

(d)

$$h[n] = \delta[n] + \delta[n-3]$$

$$H(z) = 1z^0 + z^{-3} = 1 + z^{-3}$$