PROBLEM:

We now have four ways of describing an LTI system: the difference equation with filter coefficients $\{b_k\}$; the impulse response, $h[n]$; the frequency response, $H(e^{j\hat{\omega}})$; and the system function, $H(z)$. In the following, you are given one of these representations and you must find the system function $H(z)$.

(a) $y[n] = 3(x[n] - x[n-3])$.

(b) $h[n] = -\delta[n] - \delta[n-1] - \delta[n-2] - \delta[n-3]$.

(c) $H(e^{j\hat{\omega}}) = [2j\sin(2\hat{\omega})]e^{-j3\hat{\omega}}$.

(d) $h[n] = \delta[n] + \delta[n-3]$.

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SOLUTION

(a) $y[n] = 3(x[n] - x[n-3])$

$y[n] = 3x[n] - 3x[n-3]$

$h[n] = 3\delta[n] - 3\delta[n-3]$

$H(z) = 3 - 3z^{-3}$

(b)

$h[n] = -\delta[n] - \delta[n-1] - \delta[n-2] - \delta[n-3]$

$H(z) = -1 - z^{-1} - z^{-2} - z^{-3}$

(c) $H(e^{j\hat{\omega}}) = [2j\sin(2\hat{\omega})]e^{-j3\hat{\omega}}$

We use the fact that $H(e^{j\hat{\omega}}) = H(z)$, together with the Euler identity $\sin\theta = \dfrac{e^{j\theta} - e^{-j\theta}}{2j}$.

$$H(e^{j\hat{\omega}}) = 2j\, e^{-j3\hat{\omega}}\left(\frac{e^{j2\hat{\omega}} - e^{-j2\hat{\omega}}}{2j}\right)$$

$$= e^{-j3\hat{\omega}}\left(e^{j2\hat{\omega}} - e^{-j2\hat{\omega}}\right)$$

$$= e^{-j\hat{\omega}} - e^{-j5\hat{\omega}}$$

$H(z) = z^{-1} - z^{-5}$

(d)

$h[n] = \delta[n] + \delta[n-3]$

$H(z) = 1z^{0} + z^{-3} = 1 + z^{-3}$

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