PROBLEM:

For each of the four systems given in Problem 8.1, determine the poles and zeros and make a plot of the pole-zero locations in the z-plane. Show the unit circle for reference.

Note: you can check your work with the MATLAB function `zplane()`, or the equivalent function `zz-plane()` from the class web site.

McClellan, Schafer and Yoder, *Signal Processing First*, ISBN 0-13-065562-7.

SOLUTION

(a) $H(z) = 3 - 3z^{-3}$

$$H(z) = 3 - \frac{3}{z^3} = \frac{3z^3 - 3}{z^3} = \frac{3(z^3-1)}{z^3}$$

There are 3 zeros and a triple pole at $z=0$.

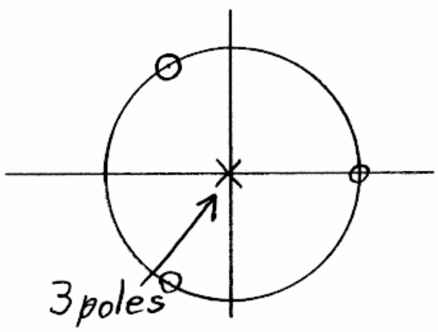
The zeros are the three roots of unity.

$1 = e^{j0}, e^{\pm j2\pi}, e^{\pm j4\pi}$ -- etc.

$z_1^3 = e^{j0}$ $\quad z_1 = 1$

$z_2^3 = e^{j2\pi}$ $\quad z_2 = e^{j\frac{2\pi}{3}} = -0.5 + j0.87$

$z_3^3 = e^{-j2\pi}$ $\quad z_3 = e^{-j\frac{2\pi}{3}} = -0.5 - j0.87$



(b) $H(z) = -1(1 + z^{-1} + z^{-2} + z^{-3})$

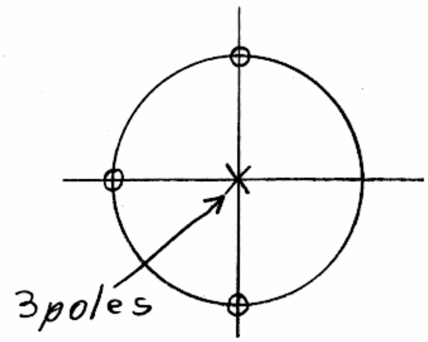
$= z^{-3}(z^3 + z^2 + z + 1)$

↖ triple pole at $z=0$

The zeros are found from the solution of

$z^3 + z^2 + z + 1 = 0$ This can be factored.

$(z+1)(z^2+1) = 0 \quad \therefore z_1 = -1 \quad z_2, z_3 = e^{j\pi/2}, e^{-j\pi/2}$



(c) $H(z) = z^{-1} - z^{-5}$

$H(z) = z^{-5}(z^4 - 1)$

↖ 5 poles at $z=0$

The zeros are found from the solution of

$z^4 - 1 = 0$ or $z^4 = 1$

$1 = e^{j0}, e^{\pm j2\pi}, e^{\pm j4\pi}$

There must be 4 roots, but e^{j0} yields only one, $z_1 = 1$

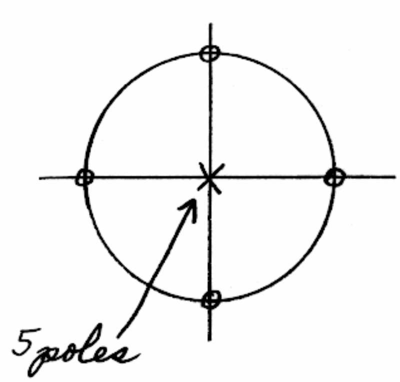
So we use $z^4 = e^{\pm j2\pi}$

$z_2 = e^{j\frac{2\pi}{4}} = e^{j\pi/2}$

$z_3 = e^{-j\pi/2}$

and $z^4 = e^{\pm j\frac{4\pi}{4}}$

$z_4 = e^{\pm j\pi} = -1.$



(d) $H(z) = 1 + z^{-3}$

$$H(z) = z^{-3}(z^3+1)$$

↑ 3 poles at $z=0$

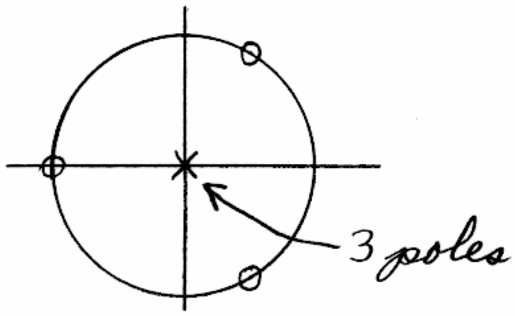
The zeros are found from $z^3+1=0$ or $z^3=-1$

$$-1 = e^{\pm j\pi},\ e^{\pm j3\pi},\ e^{\pm j5\pi} \cdots \text{etc.}$$

There are 3 roots. One is obviously at $z_1=-1$, but the procedure will yield that one.

$z^3 = e^{\pm j\pi}$ $\quad z_1 = e^{j\pi/3}$ $\quad z_2 = e^{-j\pi/3}$

$z^3 = e^{\pm j3\pi}$ $\quad z_3 = e^{\pm j\pi} = -1$



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