PROBLEM:

The diagram in Figure 1 depicts a *cascade connection* of two linear time-invariant systems; i.e., the output of the first system is the input to the second system, and the overall output is the output of the second system. In Figure 1, assume that both systems are first difference filters; i.e.,

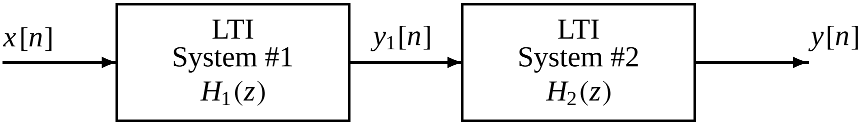


Figure 1: Cascade connection of two LTI systems.

$$y_1[n] = (x[n] - x[n-1]) \quad \text{and} \quad y[n] = (y_1[n] - y_1[n-1]).$$

(a) Determine the system function $H(z) = H_1(z)H_2(z)$ for the overall system.

(b) Plot the poles and zeros of $H(z)$ in the z-plane.

(c) From $H(z)$, determine the impulse response $h[n]$ of the overall system in Figure 1.

(d) From $H(z)$, obtain an expression for the frequency response $H(e^{j\hat{\omega}})$ of the overall cascade system.

(e) Use your result from (d) as an aid in sketching the frequency response (magnitude and phase) functions of the overall cascade system for $-\pi \leq \hat{\omega} \leq \pi$.

McClellan, Schafer and Yoder, *Signal Processing First*, ISBN 0-13-065562-7.
Prentice Hall, Upper Saddle River, NJ 07458.

SOLUTION

(a) $y_1[n] = x[n] - x[n-1]$ $\qquad$ $y[n] = y_1[n] - y_1[n-1]$

$H_1(z) = 1 - z^{-1}$ $\qquad$ $H_2(z) = 1 - z^{-1}$

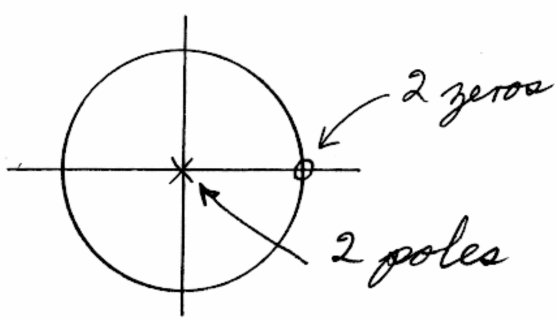
$$H(z) = H_1(z)H_2(z) = (1-z^{-1})(1-z^{-1}) = 1 - 2z^{-1} + z^{-2}$$

(b) The poles and zeros can be found most conveniently by finding them for $H_1(z)$ and $H_2(z)$, since those system functions are simpler than $H(z)$. $H_1(z)$ and $H_2(z)$ are identical, so poles and zeros will overlap.

$$H_1(z) = 1 - z^{-1} = z^{-1}(z-1) = \frac{z-1}{z}$$

$p_1 = 0$

$z_1 = 1$



(c) $H(z) = 1 - 2z^{-1} + z^{-2}$

$$h[n] = \delta[n] - 2\delta[n-1] + \delta[n-2]$$

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(d) To obtain the frequency response, we simply make the substitution $z = e^{j\hat{\omega}}$

$$\mathcal{H}(e^{j\hat{\omega}}) = 1 - 2e^{-j\hat{\omega}} + e^{-j2\hat{\omega}}$$

$$= e^{-j\hat{\omega}}\left(e^{j\hat{\omega}} - 2 + e^{-j\hat{\omega}}\right) = e^{-j\hat{\omega}}(2\cos\hat{\omega} - 2)$$

$$= 2e^{-j\hat{\omega}}(\cos\hat{\omega} - 1)$$

(e) $$H(e^{j\hat{\omega}}) = 2\underbrace{e^{-j\hat{\omega}}}_{\text{phase}}\underbrace{(\cos\hat{\omega} - 1)}_{\text{signed magnitude}}$$