PROBLEM:

The system function of a linear time-invariant filter is given by the formula

$$H(z) = (1 + z^{-1})(1 - e^{j2\pi/3}z^{-1})(1 - e^{-j2\pi/3}z^{-1})$$

(a) Write the difference equation that gives the relation between the input $x[n]$ and the output $y[n]$. Make sure that all the filter coefficients $\{b_k\}$ in your difference equation are purely real.

(b) Use multiplication of z-transform polynomials to find the output when the input is

$$x[n] = -\delta[n-2] - \delta[n-3] - \delta[n-4].$$

(c) Plot the poles and zeros of $H(z)$ in the z-plane.

(d) If the input is of the form $x[n] = Ae^{j\phi}e^{j\hat{\omega}n}$, for what values of frequency $\hat{\omega}$ will the output signal be zero for all n (i.e., $y[n] = 0$) ? Find all possible frequencies in the range $-\pi \le \hat{\omega} \le \pi$. *Hint: Take a look at the locations of the zeros of $H(z)$ as plotted in part (c).*

McClellan, Schafer and Yoder, *Signal Processing First*, ISBN 0-13-065562-7.
Prentice Hall, Upper Saddle River, NJ 07458.

SOLUTION

(a) $H(z) = (1+z^{-1})(1-e^{j2\pi/3}z^{-1})(1-e^{-j2\pi/3}z^{-1})$

These are conjugates, a hint that when multiplied out, all the coefficients will be real.

$H(z) = (1+z^{-1})(1-e^{-j2\pi/3}z^{-1}-e^{j2\pi/3}z^{-1}+e^{0}z^{-2})$

$H(z) = (1+z^{-1})(1-z^{-1}(\cos\frac{-2\pi}{3}+j\sin\frac{-2\pi}{3})-z^{-1}(\cos\frac{2\pi}{3}+j\sin\frac{2\pi}{3})+z^{-2})$

$H(z) = (1+z^{-1})(1-z^{-1}(2\cos\frac{2\pi}{3}+j0)+z^{-2})$

$H(z) = (1+z^{-1})(1+z^{-1}+z^{-2}) = 1+2z^{-1}+2z^{-2}+z^{-3}$

(b) $H(z) = 1+2z^{-1}+2z^{-2}+z^{-3}$

$x[n] = -\delta[n-2]-\delta[n-3]-\delta[n-4]$

$X(z) = -z^{-2} \quad -z^{-3} \quad -z^{-4}$

$Y(z) = X(z)\cdot H(z)$

$Y(z) = (-z^{-2}-z^{-3}-z^{-4})(1+2z^{-1}+2z^{-2}+z^{-3})$

$$\begin{array}{rrrrrr}
1 & +\,2z^{-1} & +\,2z^{-2} & +\,z^{-3} & & \\
\times & -z^{-2} & -z^{-3} & -z^{-4} & & \\
\hline
-z^{-2} & -2z^{-3} & -2z^{-4} & -z^{-5} & & \\
 & -z^{-3} & -2z^{-4} & -2z^{-5} & -z^{-6} & \\
 & & -z^{-4} & -2z^{-5} & -2z^{-6} & -z^{-7} \\
\hline
\end{array}$$

$Y(z) = -z^{-2}-3z^{-3}-5z^{-4}-5z^{-5}-3z^{-6}-z^{-7}$

$y[n] = -\delta[n-2]-3\delta[n-3]-5\delta[n-4]-5\delta[n-5]$
$\quad -3\delta[n-6]-\delta[n-7]$

(c) $H(z) = 1 + 2z^{-1} + 2z^{-2} + z^{-3}$

$H(z) = z^{-3}(z^3 + 2z^2 + 2z + 1)$

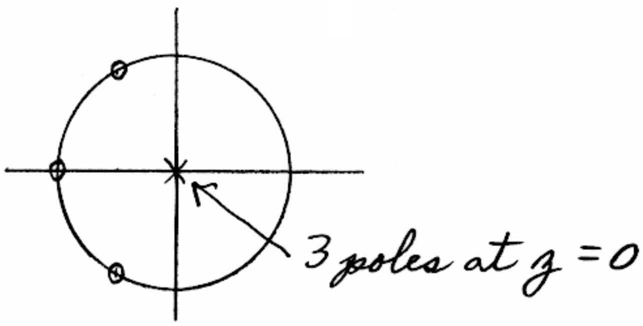
3 poles at $z = 0$

The zeros are roots of $z^3 + 2z^2 + 2z + 1 = 0$

$z_1 = -1 \qquad (z+1)(z^2 + z + 1) = 0$

Finally, the roots of $z^2 + z + 1 = 0$

$z_2, z_3 = -\frac{1}{2} \pm j\frac{\sqrt{3}}{2}$



(d) $x[n] = Ae^{j\phi}e^{j\hat{\omega}n}$

Zero	$\hat{\omega}$ (where output will be zero)
$z_1 = -1$	$\pm\pi$
$z_2 = -\frac{1}{2} + j\frac{\sqrt{3}}{2}$	$\frac{2\pi}{3}$
$z_3 = -\frac{1}{2} - j\frac{\sqrt{3}}{2}$	$\frac{-2\pi}{3}$