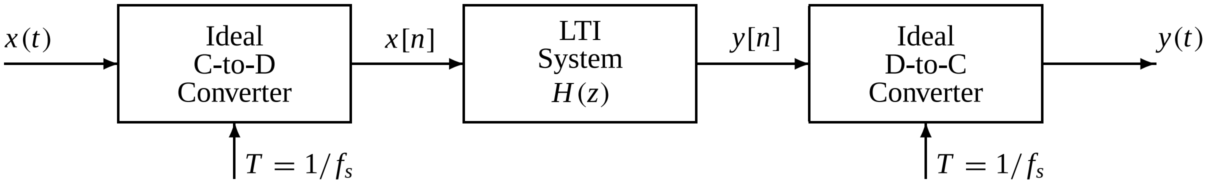
PROBLEM:

The input to the C-to-D converter in the figure below is

$$x(t) = 10 + 10\cos(5000\pi t - \pi/2) + 10\cos(12000\pi t - \pi/5)$$

The system function for the LTI system is $H(z)$, and the sampling frequency of the A/D and D/A is $f_s = 10,000$ samples/second.



Suppose that the system above corresponds to the following MATLAB program:

```
nn = 0:16000;
fs = 10000;
tn = nn/fs;
xx = 10 + 10*cos(5000*pi*tn-pi/2) + 10*cos(12000*pi*tn-pi/5);
yy = conv([3,0,0,-3],xx);
soundsc(yy,fs)
```

(a) What is the system function $H(z)$ of the system that is implemented by the `conv( )` statement?

(b) What is the frequency response of the system?

(c) Make a sketch of the spectrum of the discrete-time signal $x[n]$, the input to $H(z)$.

(d) Make a sketch of the spectrum of the discrete-time signal $y[n]$, the output from $H(z)$.

(e) Neglecting the end effects in the convolution, determine $y(t)$ that describes the signal produced by the `soundsc( )` statement.

McClellan, Schafer and Yoder, *Signal Processing First*, ISBN 0-13-065562-7.
Prentice Hall, Upper Saddle River, NJ 07458.

SOLUTION

(a) yy = conv([3,0,0 −3], xx)

The system function of the array [3,0,0, −3] is $H(z) = 3 - 3z^{-3} = 3(1-z^{-3})$

(b) $H(e^{j\hat{\omega}}) = 3(1-e^{-j3\hat{\omega}})$

$$= 3e^{-j1.5\hat{\omega}}\left(e^{j1.5\hat{\omega}} - e^{-j1.5\hat{\omega}}\right)$$

$$= 3e^{-j1.5\hat{\omega}}\, 2j \sin 1.5\hat{\omega}$$

$$= 6e^{-j1.5\hat{\omega}} e^{j\pi/2} \sin 1.5\hat{\omega}$$

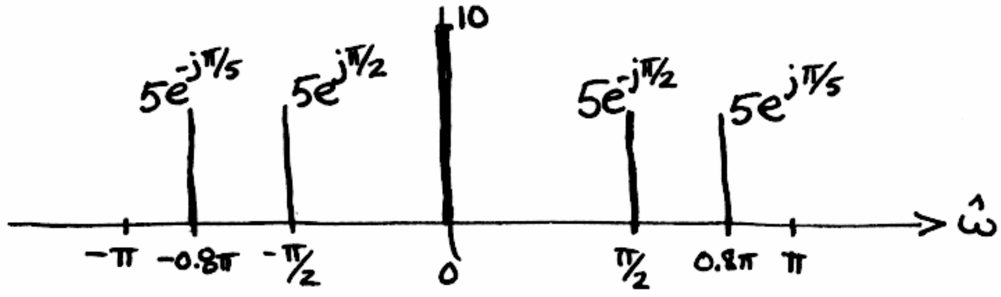
(c) $\hat{\omega} = 2\pi \dfrac{\omega}{\omega_s}$ $\hat{\omega}_1 = 2\pi\left(\dfrac{5{,}000\pi}{20{,}000\pi}\right) = \dfrac{\pi}{2}$

$$\hat{\omega}_2 = 2\pi\left(\frac{12{,}000\pi}{20{,}000\pi}\right) = \frac{6\pi}{5}$$

This is above $\hat{\omega}_{max} = \pi$

when aliased, $\hat{\omega}_2$ will appear at

$\hat{\omega}_2 = \dfrac{6\pi}{5} - 2\pi = -\dfrac{4\pi}{5}$ phase $= -\pi/5$



SPECTRUM for x[n] for $-\pi \le \hat{\omega} \le \pi$

8.5(d)

we need the frequency response @ $\hat{\omega}=0, \frac{\pi}{2}, 4\pi/5$.

$$H(e^{j0}) = H(z)\Big|_{z=1} = 3-3z^{-3}\Big|_{z=1} = 3-3=0$$

$$H(e^{j\pi/2}) = 6e^{-j1.5(\pi/2)}e^{j\pi/2}\sin(1.5\pi/2) = 4.24e^{-j\pi/4}$$

$$H(e^{j0.8\pi}) = 6e^{-j1.5(0.8\pi)}e^{j\pi/2}\sin(1.5(0.8\pi)) = 3.53e^{j0.3\pi}$$

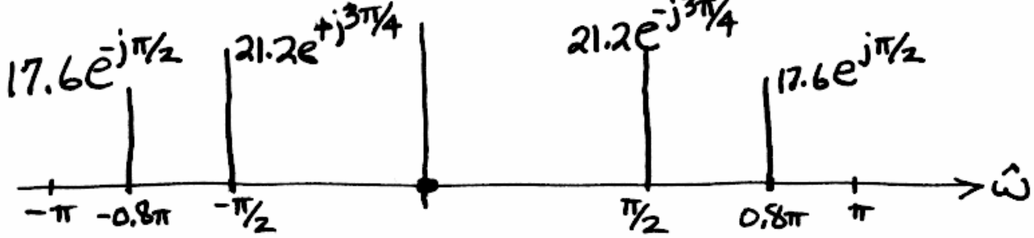
Now we multiply these complex frequency response values times the spectrum for $x[n]$.

at $\hat{\omega}=\pi/2$:

$$(4.24e^{-j\pi/4})(5e^{-j\pi/2}) = 21.2e^{-j3\pi/4}$$

at $\hat{\omega}=0.8\pi$:

$$(3.53e^{j0.3\pi})(5e^{j\pi/5}) = 17.63e^{j\pi/2}$$



8.5(e) combine the spectrum components: $(n \leftarrow f_s t)$

$$y[n] = 42.4\cos\left(\frac{\pi n}{2}-\frac{3\pi}{4}\right) + 35.2\cos(0.8\pi n + 0.5\pi)$$

$$y(t) = 42.4\cos\left(5000\pi t-\frac{3\pi}{4}\right) + 35.2\cos(8000\pi t + 0.5\pi)$$