PROBLEM:

Try your hand at expressing each of the following in a simpler form:

(a) $\delta(t-3) * [\delta(t) + 2e^{-t}\cos(5\pi t)u(t)] =$

(b) $[u(-t+3) - u(t)][\delta(t-1) + \delta(t-4)] =$

(c) $\dfrac{d}{dt}[\cos(5\pi t)u(t-1)] =$

(d) $\displaystyle\int_{-\infty}^{t} e^{-(\tau-1)}\delta(\tau-1)d\tau =$

Note: use properties of the impulse signal $\delta(t)$ and the unit-step signal $u(t)$ to perform the simplifications. For example, recall

$$\delta(t) = \frac{d}{dt}u(t) \qquad \text{where} \qquad u(t) = \begin{cases} 1 & \text{for } t \geq 0 \\ 0 & \text{for } t < 0 \end{cases}$$

Be careful to distinguish between multiplication and convolution; convolution is denoted by a "star", as in $u(t) * \delta(t-2)$.

McClellan, Schafer and Yoder, *Signal Processing First*, ISBN 0-13-065562-7.
Prentice Hall, Upper Saddle River, NJ 07458.

SOLUTION

A) $\delta(t-3) * [\delta(t) + 2e^{-t}\cos(5\pi t)u(t)]$

$= \delta(t-3) * \delta(t) + \delta(t-3) * 2e^{-t}\cos(5\pi t)\, u(t)$

$= \delta(t-3) + 2e^{-(t-3)}\cos(5\pi(t-3))\, u(t-3)$

$= \delta(t-3) + 2e^{-(t-3)}\cos(5\pi t - \pi)\, u(t-3)$

for the phase:
$-15\pi = -\pi$

B) $[u(-t+3) - u(t)][\delta(t-1) + \delta(t-4)]$

$= u(-t+3)\,\delta(t-1) + u(-t+3)\,\delta(t-4) - u(t)\,\delta(t-1) - u(t)\,\delta(t-4)$

$= u(2)\,\delta(t-1) + u(-1)\,\delta(t-4) - u(1)\,\delta(t-1) - u(4)\,\delta(t-4)$

$= \delta(t-1) - \delta(t-1) - \delta(t-4)$

$= -\delta(t-4)$

C) $\frac{d}{dt}[\cos(5\pi t)\, u(t-1)] = \cos 5\pi t \left[\frac{d\,u(t-1)}{dt}\right] + \left[\frac{d}{dt}(\cos 5\pi t)\right] u(t-1)$

$= (\cos(5\pi t))\,\delta(t-1) - 5\pi(\sin(5\pi t))\, u(t-1)$

$= \cos(5\pi)\,\delta(t-1) - 5\pi(\sin(5\pi t))\, u(t-1)$

$= -\delta(t-1) - 5\pi(\sin(5\pi t))\, u(t-1)$

D) $\int_{-\infty}^{t} e^{-(\tau-1)}\,\delta(\tau-1)\, d\tau$ using $u(t) = \int_{-\infty}^{t} \delta(\tau)\, d\tau$

$= \int_{-\infty}^{t} e^{-(1-1)}\,\delta(t-1)\, d\tau = \int_{-\infty}^{t} \delta(\tau-1)\, dt$

$= u(t-1)$