PROBLEM:

A linear time-invariant system has impulse response:

$$h(t) = \begin{cases} e^{-2t} & -1 \leq t < 1 \\ 0 & \text{otherwise} \end{cases}$$

(a) Plot $h(\tau)$ and $h(t-\tau)$ as a functions of τ for $t = -5$, 0, and 5.

(b) Is the system causal? Justify your answer.

(c) Find the output $y(t)$ when the input is $x(t) = \delta(t+5)$.

(d) Use the convolution integral to determine the output $y(t)$ when the input is

$$x(t) = \begin{cases} 1 & 0 \leq t < 5 \\ 0 & \text{otherwise.} \end{cases}$$

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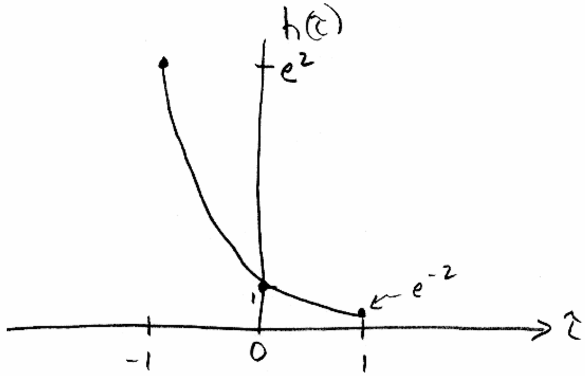
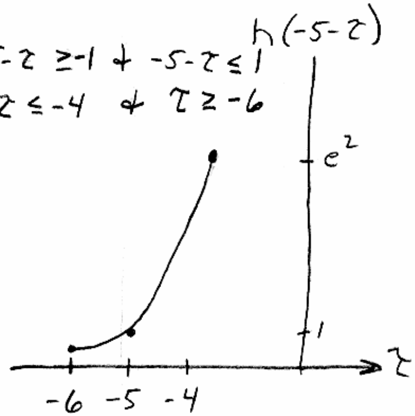
SOLUTION

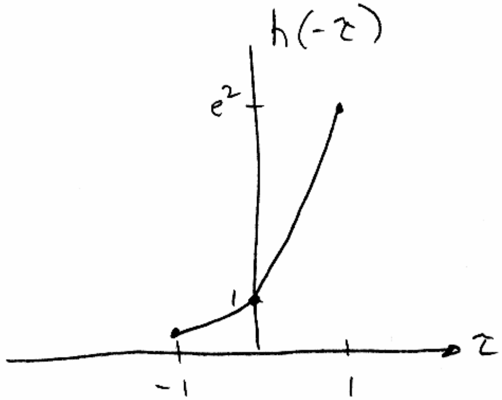
A) Plot $h(\tau)$ and $h(t-\tau)$ for $t = -5, 0, 5$

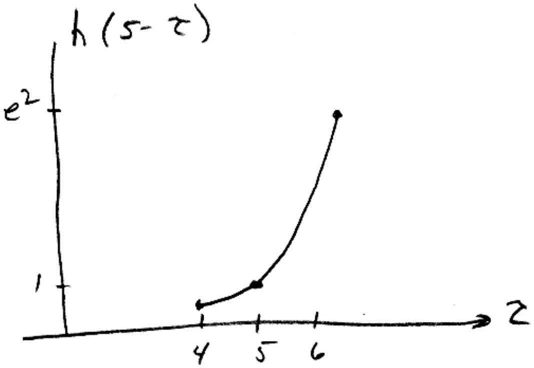


$-5-\tau \geq -1$ & $-5-\tau \leq 1$

$\tau \leq -4$ & $\tau \geq -6$







B) The system is not causal. $h(t) \neq 0$ for $t < 0$.

c) $y(t) = x(t) * h(t) = \delta(t+5) * e^{-2t} = e^{-2(t+5)}$ for $-6 \leq t \leq -4$

$$\Rightarrow y(t) = \begin{cases} e^{-10} e^{-2t} & -6 \leq t \leq -4 \\ 0 & \text{otherwise} \end{cases}$$

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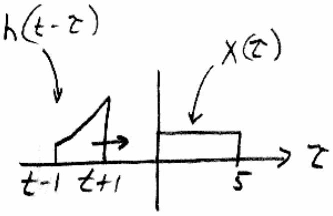
D) $x(t) = \begin{cases} 1 & 0 \le t \le 5 \\ 0 & \text{otherwise} \end{cases}$

$$y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)\,d\tau = x(t) * h(t) =$$



Interval 1 For $t+1<0$, $t<-1$, functions don't overlap

$$y(t) = 0 \quad t < -1$$

Interval 2 $t+1 \ge 0$ & $t-1 < 0 \Rightarrow -1 \le t \le 1$

$$y(t) = \int_0^{t+1} 1\, e^{-2(t-\tau)}\,d\tau = e^{-2t}\int_0^{t+1} e^{2\tau}\,d\tau$$

$$= e^{-2t}\left(\frac{e^{2(t+1)}}{2} - \frac{1}{2}\right)$$

$$y(t) = \frac{e^2}{2} - \frac{e^{-2t}}{2}$$

Interval 3 $t+1 \le 5$ & $t-1 \ge 0 \Rightarrow 1 \le t \le 4$

$$y(t) = \int_{t-1}^{t+1} 1\, e^{-2(t-\tau)}\,d\tau = e^{-2t}\int_{t-1}^{t+1} e^{2\tau}\,d\tau = e^{-2t}\left(\frac{e^{2(t+1)}}{2} - \frac{e^{2(t-1)}}{2}\right)$$

$$y(t) = \frac{e^2}{2} - \frac{e^{-2}}{2}$$

Interval 4 $t+1 \ge 5$ & $t-1 < 5 \Rightarrow 4 \le t \le 6$

$$y(t) = \int_{t-1}^{5} 1\, e^{-2(t-\tau)}\,d\tau = \frac{e^{-2(t-5)}}{2} - \frac{e^{-2}}{2}$$

Interval 5 $t-1 \ge 5 \Rightarrow t \ge 6$

$y(t) = 0$, functions don't overlap