PROBLEM:

A continuous-time system is defined by the input/output relation

$$y(t) = \int_{-\infty}^{t+4} x(\tau)d\tau.$$

(a) Determine the impulse response, $h(t)$, of this system.

(b) Is this a stable system? Explain with a proof or counter-example.

(c) Is it a causal system? Explain with a proof or counter-example.

(d) Use the convolution integral to determine the output of the system when the input is the finite-length pulse:

$$x(t) = u(t+1) - u(t-1) = \begin{cases} 1 & -1 \le t < 1 \\ 0 & \text{elsewhere.} \end{cases}$$

Plot your answer.

(e) *In this part you can double check the previous part by using the step response of the system.* It happens to be true that the output from this system is $(t+4)u(t+4)$ when the input is $u(t)$. So, use the linearity and time-invariance properties of the system to determine the output when the input is $u(t+1) - u(t-1)$. State clearly how both linearity and time-invarince were used in this solution.

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SOLUTION

A) If $y(t) = \int_{-\infty}^{t+4} x(\tau)\,d\tau$, then $h(t) = \int_{-\infty}^{t+4} \delta(\tau)\,d\tau$

$\Rightarrow h(t) = u(t+4)$

B) A system is stable if:

1. a bounded input produces bounded outputs
2. $\int_{-\infty}^{\infty} |h(t)|\,dt < \infty$

Using criteria 2, $\int_{-\infty}^{\infty} u(t+4)\,dt = \int_{-4}^{\infty} dt = \infty$

Therefore, the system is <u>unstable</u>.

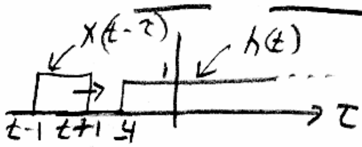
C) A system is causal if:

1. $y(t_0)$ depends only on $x(\tau)$ for $\tau \le t_0$
2. $h(t) = 0$ for $t < 0$

Using criteria 2, $h(t) = u(t+4) \neq 0$ for all $t < 0$

Therefore, the system is <u>not causal</u>.

D) $y(t) = x(t) * h(t) =$



<u>Interval 1</u>

$t+1 \le -4 \Rightarrow t \le -5$

$y(t) = 0$

<u>Interval 2</u>

$t+1 \ge -4$ & $t-1 \le -4 \Rightarrow -5 \le t \le -3$

$y(t) = \int_{-4}^{t+1} d\tau = t+5$

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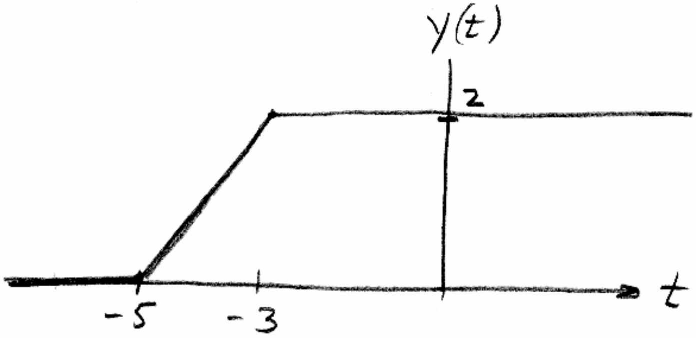
Interval 3 $\quad t-1 \geq -4 \Rightarrow t \geq -3$

$$y(t) = \int_{t-1}^{t+1} d\tau = 2$$

Plot of $y(t)$



E) If $x(t) = u(t)$ produced $y(t) = (t+4)u(t+4)$, then by time invariance $x_1(t) = u(t+1)$ produces $y_1(t) = (t+5)u(t+5)$ and $x_2(t) = u(t-1)$ produces $y_2(t) = (t+3)u(t+3)$. Using linearity (superposition), if $x(t) = x_1(t) - x_2(t)$, then $y(t) = y_1(t) - y_2(t)$,

Therefore $y(t) = (t+5)u(t+5) - (t+3)u(t+3)$

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