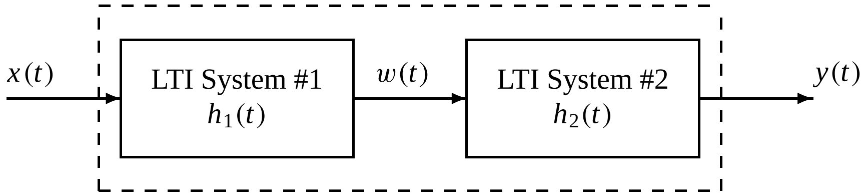
PROBLEM:



(a) In this part, assme that the first system is described by the input/output relation

$$w(t) = \int_{-\infty}^{t} x(\tau)d\tau$$

and the second system has impulse response $h_2(t) = u(t-5)$.
Find the impulse response of the overall system; i.e., find the output $y(t) = h(t)$ when the input is $x(t) = \delta(t)$.

(b) Now assume that the first system is described by the input/output relation

$$w(t) = \frac{dx(t)}{dt} + 3x(t)$$

We want to find the impulse response of the second system $h_2(t)$, so that the overall system will have an impulse response equal to an impulse, i.e., $h(t) = \delta(t)$. In this case, the impulse response of the second system has to be a one-sided exponential, $h_2(t) = Ae^{-a(t-t_0)}u(t-t_0)$. Prove that this is true by finding the unknown parameters in the formula for $h_2(t)$.
Note: this is the continuous-time version of deconvolution.

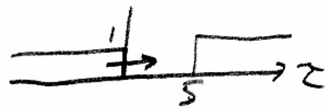
McClellan, Schafer and Yoder, *Signal Processing First*, ISBN 0-13-065562-7.

SOLUTION

$$x(t) \rightarrow \boxed{h_1(t)} \xrightarrow{w(t)} \boxed{h_2(t)} \rightarrow y(t)$$

A) If $w(t) = \int_{-\infty}^{t} x(\tau)\,d\tau$, then $h_1(t) = \int_{-\infty}^{t} \delta(\tau)\,d\tau = u(t)$

Therefore $h_{\substack{\text{overall}\\\text{system}}}(t) = h_1(t) * h_2(t) = u(t) * u(t-5)$



<u>Interval 1</u> $t < 5$

$h(t) = y(t) = 0$

<u>Interval 2</u> $t \geq 5$

$h(t) = y(t) = \int_{5}^{t} d\tau = t - 5$

B) If $w(t) = \dfrac{d\,x(t)}{dt} + 3x(t)$ and $h_{\substack{\text{overall}\\\text{system}}}(t) = \delta(t)$

$$h_{\substack{\text{overall}\\\text{system}}}(t) = \left(\frac{d\,\delta(t)}{dt} + 3\,\delta(t)\right) * h_2(t)$$

$$= \left(\frac{d\,\delta(t)}{dt} + 3\,\delta(t)\right) * e^{\alpha t} u(t) = \frac{d}{dt}\left(e^{\alpha t} u(t)\right) + 3e^{\alpha t} u(t)$$

$$= \alpha e^{\alpha t} u(t) + e^{\alpha t} \delta(t) + 3e^{\alpha t} u(t)$$

$$= (3 + \alpha) e^{\alpha t} u(t) + \delta(t)$$

Therefore for $h_{\substack{\text{overall}\\\text{system}}}(t) = \delta(t)$, then $\alpha = -3$

$$h_2(t) = e^{-3t} u(t)$$