PROBLEM:

Use the *delay property* of Fourier transforms, $x(t - t_d) \Longleftrightarrow e^{-j\omega t_d} X(j\omega)$,
to determine the Fourier transform of the following signals:

(a) $x(t) = \delta(t - 5)\cos(t)$

(b) $x(t) = \delta(t - 5) * \cos(t)$

(c) $x(t) = u(t) - u(t - 5)$

(d) $x(t) = e^{-7t}u(t) - e^{-7t}u(t - 3)$
Hint: rearrange into the following form: $x(t) = e^{-7t}u(t) - \beta e^{-7(t-3)}u(t - 3)$

(e) $x(t) = [u(t) - u(t - 5)]\cos(50\pi t)$
In this part, use *frequency shifting property:* $x(t)e^{j\omega_c t} \Longleftrightarrow X(j(\omega - \omega_c))$,

McClellan, Schafer and Yoder, *Signal Processing First*, ISBN 0-13-065562-7.
Prentice Hall, Upper Saddle River, NJ 07458.

SOLUTION

a) $x(t) = \delta(t-5)\cos(t) = \delta(t-5)[\cos(5) \cong 0.284]$

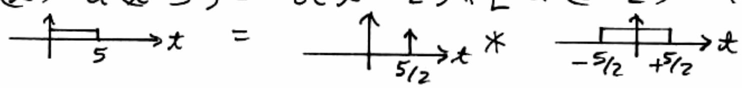
MULTIPLY $\mathfrak{F}\{\cos(5)\,\delta(t-5)\} = e^{-j5\omega}\cos(5)\,\mathfrak{F}\{\delta(t)\} \cong e^{-j5\omega}\cdot 0.284$

b) $x(t) = \delta(t-5) * \cos(t) =$

CONVOLVE $\underbrace{\mathfrak{F}\{\delta(t-5) * \cos(t)\} = \mathfrak{F}\{\delta(t-5)\}\cdot\mathfrak{F}\{\cos(t)\}}_{\text{FOURIER TRANSFORM PROPERTY}}$

$= e^{-j5\omega}\cdot 1\cdot[\delta(\omega+1) + \delta(\omega-1)]\cdot\pi$

c) $x(t) = u(t) - u(t-5) = \delta(t-\frac{5}{2}) * [u(t+\frac{5}{2}) - u(t-\frac{5}{2})]$



$X(\omega) = \mathfrak{F}\{\delta(t-\frac{5}{2})\}\cdot\mathfrak{F}\{u(t+5/2) - u(t-5/2)]$

$= e^{-j\omega\frac{5}{2}}\cdot\frac{\sin(\omega\, 5/2)}{\omega/2}$

d) $x(t) = e^{-7t}u(t) - [e^{-t7}u(t-3) = e^{-(t-3)7}u(t-3)e^{-21}]$

$= e^{-7t}[u(t) - u(t-3)] \leftarrow$ GOOD TRY BUT $\mathfrak{F}\{e^{-7t}\}$ EXPLODES

$= e^{-7t}u(t) - \delta(t-3) * [e^{-(t+3)7}u(t) = e^{-21}e^{-t7}u(t)]$

$X(j\omega) = \frac{1}{7+j\omega} - \frac{e^{-j\omega 3}\cdot e^{-21}}{7+j\omega} = \frac{1 - e^{-j\omega 3}(e^{-21} \simeq 10^{-9})}{7+j\omega} \simeq \frac{1}{7+j\omega}$

e) $x(t) = [u(t) - u(t-5)]\cdot\cos(50\pi t)$

$= \{\delta(t-\frac{5}{2}) * [u(t+\frac{5}{2}) - u(t-\frac{5}{2})]\}\cdot\cos(50\pi t)$

$X(j\omega) = \left[e^{-j\omega\frac{5}{2}}\cdot\frac{\sin(\omega\, 5/2)}{\omega/2}\right] * [\delta(\omega+50\pi) + \delta(\omega-50\pi)]\pi$

$= \pi\left[e^{-j\frac{5}{2}(\omega+50\pi)}\frac{\sin[(\omega+50\pi)\frac{5}{2}]}{(\omega+50\pi)/2} + e^{-j\frac{5}{2}(\omega-50\pi)}\frac{\sin[(\omega-50\pi)\frac{5}{2}]}{(\omega-50\pi)/2}\right]$

$= \pi e^{j\pi}\left[e^{-j\frac{5\omega}{2}}\frac{\sin[(\omega+50\pi)\frac{5}{2}]}{\frac{\omega}{2}+25\pi} + e^{-j\frac{5\omega}{2}}\frac{\sin[(\omega-50\pi)\frac{5}{2}]}{\frac{\omega}{2}-25\pi}\right]$

$= \pi e^{j(\pi-\frac{5\omega}{2})}\left[\frac{\sin(\frac{5}{2}\omega+\pi)}{\frac{\omega}{2}+25\pi} + \frac{(\sin\frac{5}{2}\omega+\pi)}{\frac{\omega}{2}-25\pi}\right]$