PROBLEM:

A continuous-time linear time-invariant system has impulse response

$$h(t) = \delta(t-3) - e^{-7(t-3)}u(t-3).$$

(a) Determine the frequency response $H(j\omega)$ of the system. Simplify your answer so that one part becomes a rational function with powers of $(j\omega)$ in the numerator and denominator, and the other part can be associated with the delay in $h(t)$.

(b) Plot the magnitude squared, $|H(j\omega)|^2 = H(j\omega)H^*(j\omega)$, versus ω. Likewise, plot the phase $\angle H(j\omega)$ as a function of ω.
Hint: use MATLAB or a calculator for this.

(c) Use superposition to find the output due to an input that is the sum of three terms:

$$x(t) = 7 + 7\cos(7t + \tfrac{1}{2}\pi) + \delta(t-7).$$

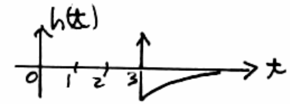
Hint: Use the easiest method (impulse response or frequency response) to find the output due to each component of the input.

McClellan, Schafer and Yoder, *Signal Processing First*, ISBN 0-13-065562-7.

SOLUTION

a) $h(t) = \delta(t-3) - e^{-7(t-3)}u(t-3) =$

$= \delta(t-3) * [\delta(t) - e^{-7t}u(t)]$



$H(j\omega) = e^{-j3\omega} \cdot \left[1 - \frac{1}{7+j\omega}\right] = e^{-j\omega 3}\left(\frac{6+j\omega}{7+j\omega}\right)$

b) SEE MATLAB PLOTS

c) $x(t) = 7 + 7\cos(7t + \frac{1}{2}\pi) + \delta(t-7)$

$\underbrace{\omega = 0 = DC \qquad \omega = 7\,[rad/sec]}_{\text{WORK IN FREQUENCY DOMAIN}}$ $\qquad \underbrace{\text{All frequencies}}_{\text{Delay in time} \Leftrightarrow \text{CONV WITH IMPULSE}}$

$y(t) = H(0)\cdot 7 + H(7)\cdot 7\cos(7t + \frac{1}{2}\pi + \angle H(7)) + \underbrace{\delta(t-7) * h(t)}$

$H(0) = \frac{6}{7}$

660.16π

$H(7) \cong 0.9313 \angle(-0.6601\pi\ [rad])$

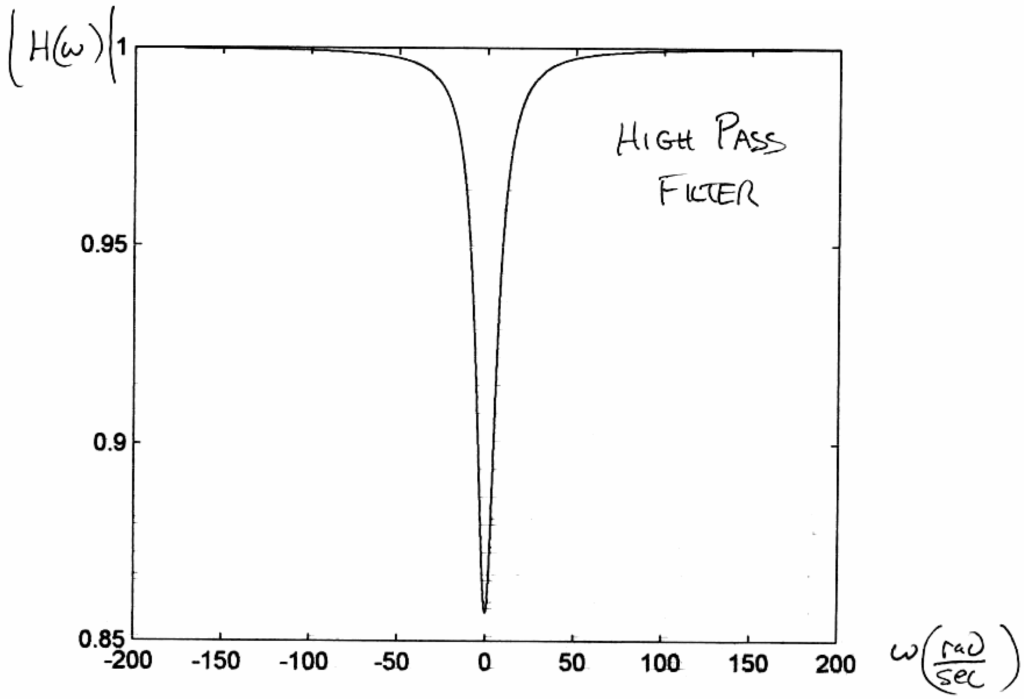
$\delta(t-7) * [\delta(t-3) - e^{-7(t-3)}u(t-3)]$

$= \delta(t-10) - e^{-7(t-10)}u(t-10)$

$y(t) = 6 + 6.52\cos(7t - 0.16\pi) + \delta(t-10) - e^{-7(t-10)}u(t-10)$

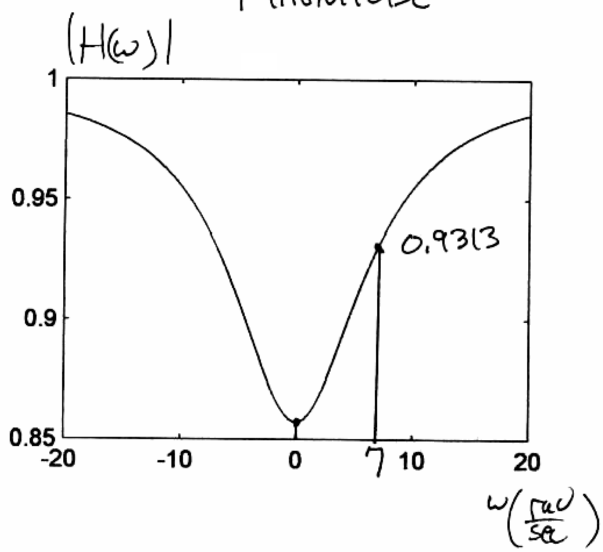
PLOTS

$$X \longrightarrow \boxed{H(\omega) = \tilde{\chi}} \longrightarrow Y$$



$$H(\omega) = \frac{(6+j\omega)}{(7+j\omega)} e^{-j\omega 3}$$

MAGNITUDE



ANGLE