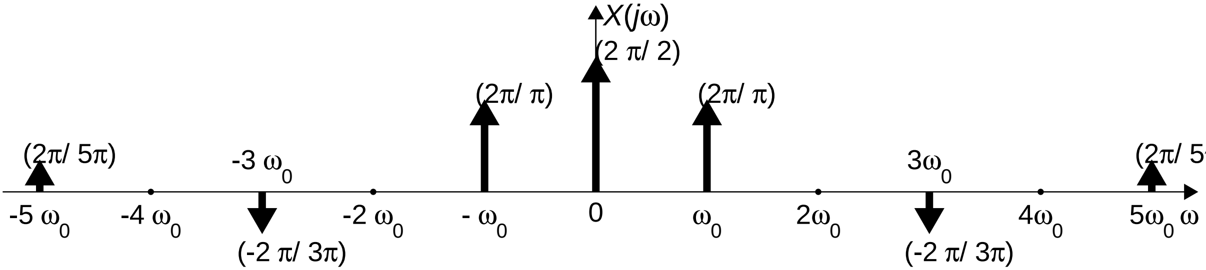
PROBLEM:

The periodic input to a LTI system $H(j\omega)$ has Fourier transform $X(j\omega)$ as defined below:



where the dark arrows denote impulses.

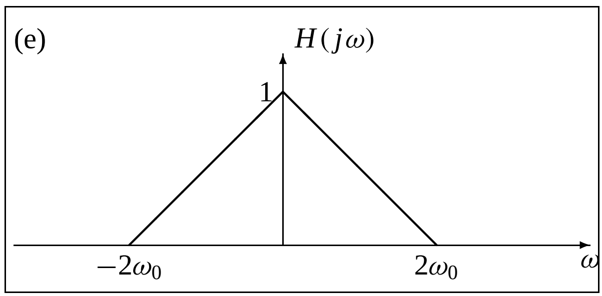
There are six possible filters: each one is described by one of the equations or graphs below. In each case determine the output signal $y(t)$. Justify your answer by giving a derivation, or by explaining how you used the Fourier transform filtering property to get $y(t)$. Give a simple formula for $y(t)$.
In most cases the output will be a sum of sinusoids, but it is permissible to write $y(t)$ in terms of $x(t)$, e.g., one of the answers can be written in the form $y(t) = Ax(t - t_d)$ and another one is $y(t) = x(t) - c$. Of course, you have to find the parameters, t_d, A and c.

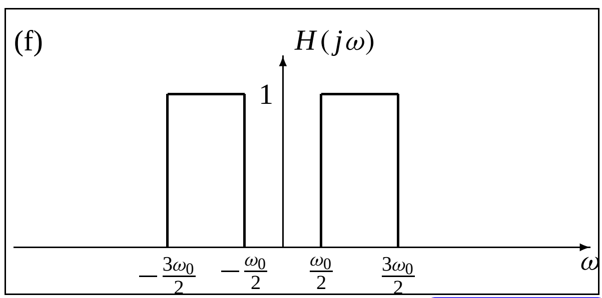
(a) $H(j\omega) = \begin{cases} 1 & |\omega| < \omega_0/2 \\ 0 & |\omega| > \omega_0/2 \end{cases}$

(b) $H(j\omega) = 500e^{-j2\omega/3}$

(c) $H(j\omega) = \begin{cases} e^{-j2\omega/3} & |\omega| < 3\omega_0/2 \\ 0 & |\omega| > 3\omega_0/2 \end{cases}$

(d) $H(j\omega) = \begin{cases} 0 & |\omega| < \omega_0/2 \\ 1 & |\omega| > \omega_0/2 \end{cases}$





McClellan, Schafer and Yoder, *Signal Processing First*, ISBN 0-13-065562-7.

SOLUTION

a) $Y(j\omega) = H(j\omega)\, X(j\omega)$

$$= \frac{2\pi}{2}\,\delta(\omega) \quad \longleftrightarrow \quad y(t) = \frac{1}{2}$$

b) $Y(j\omega) = 500\, e^{-j2\omega/3}\, X(j\omega)$

$\updownarrow$ delay property

$$y(t) = 500\, x\left(t - \frac{2}{3}\right)$$

c) $$Y(j\omega) = \left[\begin{cases} e^{-j2\omega/3} & |\omega| < 3\omega_0/2 \\ 0 & |\omega| > 3\omega_0/2 \end{cases}\right] X(j\omega)$$

$$= \underbrace{e^{-j2\omega/3}}_{\text{delay prop.}} \cdot \underbrace{\left[\begin{cases} 1 & |\omega| < 3\omega_0/2 \\ 0 & |\omega| > 3\omega_0/2 \end{cases}\right]}_{\text{this filters out all frequencies } \omega > \frac{3\omega_0}{2}} X(j\omega)$$

(∴ only DC and freq. ω_0 are left)

$$y(t) = \text{delayed } \left(+\tfrac{2}{3}\right) \text{ version of } \left\{\frac{1}{2} + \frac{2}{\pi}\cos\omega_0 t\right\}$$

$$= \frac{1}{2} + \frac{2}{\pi}\cos\omega_0\left(t - \frac{2}{3}\right)$$

d) $$Y(j\omega) = \left[\begin{cases} 0 & |\omega| < \omega_0/2 \\ 1 & |\omega| > \omega_0/2 \end{cases}\right] X(j\omega)$$

filters out (only) the DC component of $x(t)$

$$y(t) = x(t) - \frac{1}{2}$$

e) Passes the DC without attenuation
Attenuates $\left(\frac{2}{\pi}\cos\omega_0 t\right)$ by factor 1/2
Blocks the rest

$$\Bigg\}\quad y(t) = \frac{1}{2} + \frac{1}{\pi}\cos\omega_0 t$$

f) Passes ONLY the $\frac{2}{\pi}\cos\omega_0 t$ component : $y(t) = \frac{2}{\pi}\cos\omega_0 t$