PROBLEM:

The system function $H(z)$ and the impulse response $h[n]$ are two ways to define a LTI system. Use z-transform to answer the following:

(a) Find the system function for $h_a[n] = u[n] - u[n-5]$.
Use the z-transform of $u[n]$ to express your answer as a ratio of polynomials in z^{-1}. Then simplify to get a polynomial in z^{-1} (i.e., no denominator). Is this an FIR or IIR filter?

(b) Find the system function for $h_b[n] = (\frac{1}{2})^n u[n] + (-\frac{1}{2})^n u[n]$.
Express your answer as: (1) a sum of two first-order rational functions; and (2) a ratio of polynomials in z^{-1} (one numerator over one denominator).

(c) Determine the impulse response when the system function is: $H_c(z) = \dfrac{1 - z^{-2}}{1 + 0.5z^{-1}}$.

(d) Determine the impulse response when $H_d(z) = \dfrac{1 + z^{-1}}{(1 - \frac{1}{2}z^{-1})(1 + 0.75z^{-1})}$.
Hint: write $H_d(z)$ as a sum of first-order rational functions.

(e) Determine the impulse response when $H_e(z) = 1 + 2z^{-2} + 4z^{-4} - 6z^{-6} - 8z^{-8}$.

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SOLUTION

(a) $h_a[n] = u[n] - u[n-5]$

$$H_a(z) = \frac{1}{1-z^{-1}} - \frac{z^{-5}}{1-z^{-1}}$$ SEE Z-TRANSFORM TABLE

$$= \frac{(1-z^{-5}) = \cancel{(1-z^{-1})}(1+z^{-1}+z^{-2}+z^{-3}+z^{-4})}{\cancel{(1-z^{-1})}}$$

$$= 1+z^{-1}+z^{-2}+z^{-3}+z^{-4} \Longrightarrow \text{FIR (no poles)}$$

(b) $h_b[n] = \left(\frac{1}{2}\right)^n u[n] + \left(-\frac{1}{2}\right)^n u[n]$

$$= \frac{1}{1-\frac{1}{2}z^{-1}} + \frac{1}{1-(-\frac{1}{2}z^{-1})} = \frac{2}{1-\frac{1}{4}z^{-2}}$$

(c) $$H_c(z) = \frac{1-z^{-2}}{1+\frac{1}{2}z^{-1}} = \frac{1}{1+\frac{1}{2}z^{-1}} - \frac{z^{-2}}{1+\frac{1}{2}z^{-1}}$$

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(d)

$$H_d(z) = \frac{1+z^{-1}}{(1-\frac{1}{2}z^{-1})(1+\frac{3}{4}z^{-1})}$$

$$= \frac{A}{(1-\frac{1}{2}z^{-1})} + \frac{B}{(1+\frac{3}{4}z^{-1})}$$

SEE "PARTIAL FRACTION EXPANSION"

$$\boxed{A = \frac{6}{5} \qquad B = -\frac{1}{5}}$$

$$= \frac{\frac{6}{5}}{1-\frac{1}{2}z^{-1}} + \frac{-\frac{1}{5}}{1+\frac{3}{4}z^{-1}}$$

$$\hookrightarrow h[n] = \frac{6}{5}\left(\frac{1}{2}\right)^n u[n] - \frac{1}{5}\left(-\frac{3}{4}\right)^n u[n]$$

(e)

$$H_e(z) = 1 + 2z^{-2} + 4z^{-4} - 6z^{-6} - 8z^{-8}$$

$$= 1 + 0z^{-1} + 2z^{-2} + 0z^{-3} + 4z^{-4} + 0z^{-5} - 6z^{-6} + 0z^{-7} - 8z^{-8}$$

$$\hookrightarrow h[n] = \begin{bmatrix} 1 & 0 & 2 & 0 & 4 & 0 & -6 & 0 & -8 \end{bmatrix}$$