PROBLEM:

An LTI system has the following system function:

$$H(z) = \frac{1 - z^{-2}}{1 + 0.5z^{-1}}.$$

The following questions cover most of the ways available for analyzing IIR discrete-time systems.

(a) Plot the poles and zeros of $H(z)$ in the z-plane.

(b) Determine the difference equation that is satisfied by the general input $x[n]$ and the corresponding output $y[n]$ of the system.

(c) Use z-transforms to determine the impulse response $h[n]$ of the system; i.e., the output of the system when the input is $x[n] = \delta[n]$.

(d) Determine an expression for the frequency response $H(e^{j\hat{\omega}})$ of the system.

(e) Use the frequency response function to determine the output $y_1[n]$ of the system when the input is

$$x_1[n] = 2\cos(\pi n) \qquad -\infty < n < \infty.$$

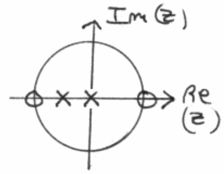
(f) Use the z-transform to determine the output $y_2[n]$ when the input is

$$x_2[n] = 2\cos(\pi n)u[n] = \begin{cases} 2(-1)^n & n \geq 0 \\ 0 & n < 0. \end{cases}$$

McClellan, Schafer and Yoder, *Signal Processing First*, ISBN 0-13-065562-7.
Prentice Hall, Upper Saddle River, NJ 07458. © 2003 Pearson Education, Inc.

SOLUTION

(a) $$H(z) = \frac{1-z^{-2}}{1+\frac{1}{2}z^{-1}} = \frac{(1+z^{-1})(1-z^{-1})}{1+\frac{1}{2}z^{-1}} \Rightarrow \begin{array}{l}\text{zeros at } \pm 1 \\ \text{poles at } 0, -\frac{1}{2}\end{array}$$



(b) $$y[n] = -\frac{1}{2}y[n-1] + x[n] - x[n-2]$$

(c) $$H(z) = \frac{1}{1+\frac{1}{2}z^{-1}} + \frac{-z^{-2}}{1+\frac{1}{2}z^{-1}} \leftarrow \text{2 (delays)}$$

$$\leftrightarrow \quad \left(-\tfrac{1}{2}\right)^n u[n] - \left(-\tfrac{1}{2}\right)^{(n-2)} u[n-2] = h[n]$$

(d) $$H(e^{j\omega}=z) = \frac{1-e^{-j2\hat{\omega}}}{1+\frac{1}{2}e^{-j\hat{\omega}}}$$

(e) $$x_1[n] = 2\cos(\pi n) \Rightarrow \hat{\omega} = \pi$$

$$y[n] = x[n]\cdot\{H(\hat{\omega}=\pi)=0\} = 0$$

(f) $$x_2[n] = 2(-1)^n u[n] \leftrightarrow \frac{2}{1+z^{-1}} = X_2(z)$$

$$Y(z) = X_2(z)\cdot H(z) = \frac{2}{\cancel{(1+z^{-1})}}\cdot\frac{\cancel{(1+z^{-1})}(1-z^{-1})}{1+\frac{1}{2}z^{-1}}$$

$$= \frac{2(1-z^{-1})}{1+\frac{1}{2}z^{-1}} = \frac{2}{1+\frac{1}{2}z^{-1}} - \frac{2z^{-1}}{1+\frac{1}{2}z^{-1}}$$

$$\updownarrow \qquad\qquad \updownarrow$$

$$y[n] = 2\left(-\tfrac{1}{2}\right)^n u[n] - 2\left(-\tfrac{1}{2}\right)^{n-1} u[n-1]$$

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