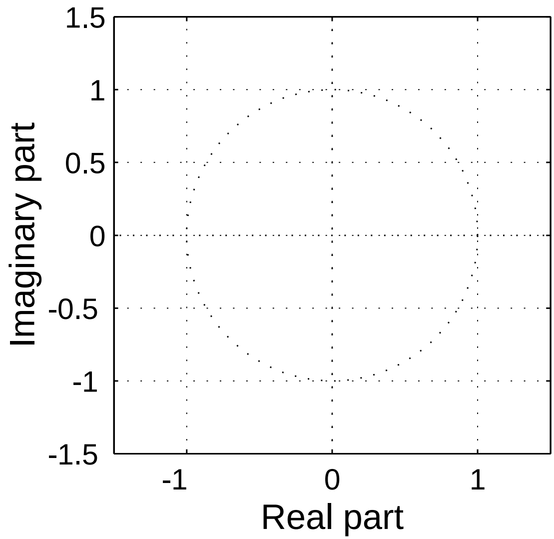
PROBLEM:

A discrete-time system (FIR filter) is defined by the following z-transform system function:

$$H(z) = (1 + 0.5z^{-1})(1 - e^{j\pi/4}z^{-1})(1 - e^{-j\pi/4}z^{-1})$$

(a) Write down the difference equation that is satisfied by the input $x[n]$ and output $y[n]$ of the system. Give the numerical values of all filter coefficients.

(b) Determine *all* the zeros of $H(z)$ and plot them in the z-plane.



(c) If the input is of the form $x[n] = s[n] + A\cos(\hat{\omega}_0 n + \phi)$, where $s[n]$ is a speech signal, for what value of frequency $\hat{\omega}_0$ (in the range $0 < \hat{\omega}_0 < \pi$) will the filter completely remove the sinusoidal component? **EXPLAIN your answer.**

McClellan, Schafer and Yoder, *Signal Processing First*, ISBN 0-13-065562-7.
Prentice Hall, Upper Saddle River, NJ 07458.

SOLUTION

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(a) Write down the difference equation that is satisfied by the input $x[n]$ and output $y[n]$ of the system. Give the numerical values of all filter coefficients.

Mutiply the factors:

$$(1+0.5z^{-1})(1-\underbrace{(e^{j\pi/4}+e^{-j\pi/4})}_{2\cos(\pi/4)=\sqrt{2}}z^{-1}+z^{-2})$$

$$H(z) = 1 + \underbrace{(0.5-\sqrt{2})}_{-0.9142}z^{-1} + \underbrace{(1-\tfrac{1}{2}\sqrt{2})}_{0.2929}z^{-2} + \tfrac{1}{2}z^{-3}$$

$H(z) = \sum b_k z^{-k}$

$$y[n] = x[n] - 0.9142x[n-1] + 0.2929x[n-2] + 0.5x[n-3]$$

(b) Determine *all* the zeros of $H(z)$ and plot them in the z-plane.

Each factor gives one zero: $(1-az^{-1}) = \frac{z-a}{z}$

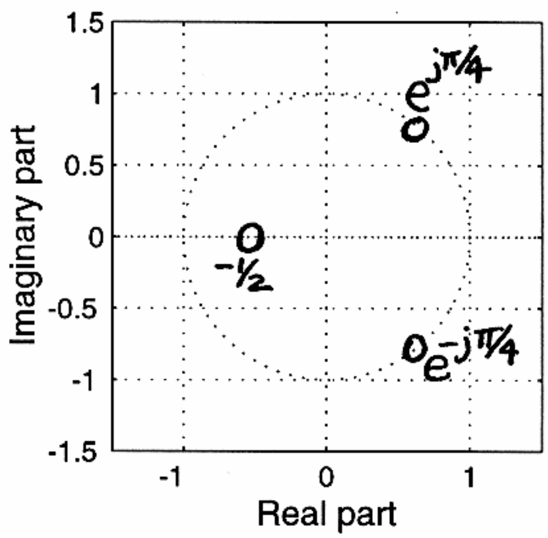
So the zero is @ $z=a$.

Zeros:

$z = -0.5$

$= e^{j\pi/4} = \frac{\sqrt{2}}{2} + j\frac{\sqrt{2}}{2}$

$= e^{-j\pi/4}$



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A zero on the unit circle will create a zero in the frequency response. Zero @ $z = e^{j\pi/4}$ makes $H(e^{j\pi/4}) = 0$, so $\hat{\omega}_0 = \pi/4$ would be "nulled out."

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