PROBLEM:

Consider a continuous-time signal

$$x(t) = A\cos(\omega_0 t + \phi)$$

We know that this signal is periodic with period $T_0 = 2\pi/\omega_0$; i.e. $x(t + T_0) = x(t)$ for all t. Now suppose that $x(t)$ is sampled to obtain the sequence

$$x[n] = x(nT_s) = A\cos(\omega_0 nT_s + \phi) = A\cos(\hat{\omega}_0 n + \phi)$$

where $\hat{\omega}_0 = \omega_0 T_s$.

Now a discrete-time signal is periodic with period N if $x[n+N] = x[n]$ for all n, where N is necessarily an integer.

(a) Will $x[n]$ be periodic for all possible sampling rates? If not, what condition on T_s will ensure that $x[n]$ is periodic with period N?

(b) If $\omega_0 = 2000\pi$, what value of T_s will result in a periodic sequence with period $N = 100$?

McClellan, Schafer and Yoder, *Signal Processing First*, ISBN 0-13-065562-7.

SOLUTION

(a) NO

$$A\cos(\omega_0 n T_s + \phi) = A\cos[\omega_0 (n+N) T_s + \phi]$$

$$= A\cos[\omega_0 n T_s + \underbrace{\omega_0 N T_s}_{2\pi} + \phi]$$

$$\omega_0 N T_s = 2\pi$$

$$T_s = \frac{2\pi}{\omega_0 N}$$

(b)

$$T_s = \frac{2\pi}{2000\pi(100)} = 10^{-5}\ \text{sec.}$$

McClellan, Schafer, and Yoder, *Signal Processing First*, ISBN 0-13-065562-7.