PROBLEM:

We have developed several concepts that are useful in solving problems involving LTI systems. The main concepts are the *difference equation*, the *impulse response*, the *system function*, and the *frequency response* function. You need to be able to go back and forth among these different mathematical representations of the LTI system because, as simple as it seems, the z-transform is *not* always the best tool for solving these problems. Indeed for a specific problem, one of these representations may be more convenient than the others, or we may need to use more than one of these representations in solving a given problem. The following is a simple problem that might be posed about an LTI system:

> *Given the input sequence $x[n]$ find the output sequence $y[n]$ of a 5-point running average filter for all values of n.*

The following is a partial list of possible approaches to solving this problem:

1. Use the difference equation representation of the system to compute the output $y[n]$ for all required values of n.

2. Multiply the z-transform of the input by the system function and determine $y[n]$ as the inverse z-transform of $Y(z)$.

3. Break the input into a sum of complex exponential signals, use the frequency response function to determine the output due to each complex exponential signal separately, and finally, add the individual outputs together to get $y[n]$.

In each of these solution methods you would use one or more of the basic representations of the 5-point running average filter. Which method is easiest will have a lot to do with the nature of the input signal. This may require that you convert a given representation of the system into one of the other forms. For example, if you are given the difference equation and you want to use approach #2, you will have to determine the system function $H(z)$ from the difference equation coefficients.

Now in each of the following cases, the input will be given. In each case, determine which representation of the system and which of the above approaches will lead to the easiest solution of the problem, and detail the steps in using that approach to solve the problem. For example, if you choose approach #2 to solve the problem, your answer should be something like the following:

Step 1 Find $X(z)$, the z-transform of $x[n]$.

Step 2 Find $H(z)$, the system function of the 5-point running averager.

Step 3 Multiply $X(z)H(z)$ to get $Y(z)$.

Step 4 Take the inverse z-transform of $Y(z)$ to get $y[n]$.

Now here are some possible inputs. In each case, state which of the above (#1, #2, or #3) approaches you would use. There may not be a clear cut answer. Give the approach that you *think* will yield the solution with least effort. Outline your approach to solving the problem of finding the output of the 5-point moving averager. **You do not have to actually find the output—just tell how you would solve it in a step-by-step procedure described as illustrated above.**

(a) $x[n]$ is a sampled speech signal. It is represented by a vector of 10000 numbers.

(b) $x[n] = 4\cos(0.1\pi n + \pi/2) + 3\cos(0.4\pi n - \pi)$ for $-\infty < n < \infty$.

(c) $x[n] = \begin{cases} 1 & 0 \le n \le 10 \\ 0 & \text{otherwise.} \end{cases}$

(d) $x[n] = 10\delta[n-50]$.

(e) $x[n] = 10\delta[n-50] + 4\cos(0.1\pi n + \pi/2) + 3\cos(0.4\pi n - \pi)$ for $-\infty < n < \infty$.

McClellan, Schafer and Yoder, *Signal Processing First*, ISBN 0-13-065562-7.
Prentice Hall, Upper Saddle River, NJ 07458. © 2003 Pearson Education, Inc.

SOLUTION

a) Obtain the difference equation describing the filter:

$$y[n] = \frac{1}{5}\sum_{k=0}^{4} x[n-k]$$

→ Use this equation to compute the values of $y[n]$ from $x[n]$.

b) Compute the output generated by each sinusoidal term:

$$4\cos\left(0.1\pi n + \frac{\pi}{2}\right) \Rightarrow 4\,\left|\mathcal{H}(0.1\pi)\right| \cdot$$

$$\cos\left(0.1\pi n + \frac{\pi}{2} + \angle\mathcal{H}(0.1\pi)\right)$$

$$3\cos\left(0.4\pi n - \pi\right) \Rightarrow 3\,\left|\mathcal{H}(0.4\pi)\right| \cdot$$

$$\cos\left(0.4\pi n - \pi + \angle\mathcal{H}(0.4\pi)\right)$$

→ Then add the two output terms to obtain the overall output generated by $x[n]$.

McClellan, Schafer, and Yoder, *Signal Processing First*, ISBN 0-13-065562-7.

c) Compute $X(z) = \sum_{k=0}^{10} z^{-k}$

$\rightarrow$ Compute $H(z) = \frac{1}{5}\sum_{k=0}^{4} z^{-k}$

$\rightarrow$ $Y(z) = H(z)\,X(z)$

$\rightarrow$ Compute the inverse z-transform of $Y(z)$ to get $y[n]$.

d) Use linearity and time-invariance:

$$\delta[n] \Rightarrow h[n] = \frac{1}{5}\sum_{k=0}^{4} \delta[n-k]$$

so $10\,\delta[n-50] \Rightarrow 10\,h[n-50]$

e) Use linearity:

$$10\,\delta[n-50] \Rightarrow 10\,h[n-50]$$

$4\cos(0.1\pi n + \frac{\pi}{2}) + 3\cos(0.4\pi n - \pi) \Rightarrow$ the output computed in part (b).

$\rightarrow$ Add all the output terms to obtain the overall output generated by $x[n]$.