PROBLEM:

We now have four ways of describing an LTI system: the difference equation; the impulse response, $h[n]$; the frequency response, $H(e^{j\hat{\omega}})$; and the system function, $H(z)$. In the following, you are given one of these representations and you must find the other three.

(a) $y[n] = (x[n] + 2x[n-2] + +x[n-4])$.

(b) $h[n] = \delta[n] + \delta[n-1] + \delta[n-2] + \delta[n-3] + \delta[n-4]$.

(c) $H(e^{j\hat{\omega}}) = [1 + \cos(2\hat{\omega})]e^{-j\hat{\omega}3}$.

(d) $H(z) = 1 - 2z^{-2} + z^{-4} + z^{-7}$.

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SOLUTION

a) $h[n] = \delta[n] + 2\delta[n-2] + \delta[n-4]$

(by definition of impulse response, $h[n]$ is the output generated by $x[n] = \delta[n]$)

$$\mathcal{H}(\hat{\omega}) = 1 + 2e^{-j2\hat{\omega}} + e^{-j4\hat{\omega}}$$

$$H(z) = 1 + 2z^{-2} + z^{-4}$$

b) $y[n] = x[n] + x[n-1] + x[n-2] + x[n-3] + {}$
$\quad + x[n-4]$

$$\mathcal{H}(\hat{\omega}) = 1 + e^{-j\hat{\omega}} + e^{-j2\hat{\omega}} + e^{-j3\hat{\omega}} + e^{-j4\hat{\omega}}$$

$$H(z) = 1 + z^{-1} + z^{-2} + z^{-3} + z^{-4}$$

c) $\mathcal{H}(\hat{\omega}) = \left(1 + \dfrac{e^{j2\hat{\omega}} + e^{-j2\hat{\omega}}}{2}\right) e^{-j3\hat{\omega}} =$

$$= \frac{1}{2}e^{-j\hat{\omega}} + e^{-j3\hat{\omega}} + \frac{1}{2}e^{-j5\hat{\omega}}$$

$$H(z) = \frac{1}{2}z^{-1} + z^{-3} + \frac{1}{2}z^{-5}$$

$$h[n] = \frac{1}{2}\delta[n-1] + \delta[n-3] + \frac{1}{2}\delta[n-5]$$

$$y[n] = \frac{1}{2}x[n-1] + x[n-3] + \frac{1}{2}x[n-5]$$

d) $\mathcal{H}(\hat{\omega}) = 1 - 2e^{-j2\hat{\omega}} + e^{-j4\hat{\omega}} + e^{-j7\hat{\omega}}$

$$h[n] = \delta[n] - 2\delta[n-2] + \delta[n-4] + \delta[n-7]$$

$$y[n] = x[n] - 2x[n-2] + x[n-4] + x[n-7]$$

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