



PROBLEM:

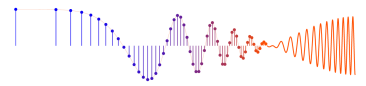
We now have four ways of describing an LTI system: the difference equation; the impulse response, $h[n]$; the frequency response, $H(e^{j\hat{\omega}})$; and the system function, $H(z)$. In the following, you are given one of these representations and you must find the other three.

(a) $y[n] = (x[n] + 2x[n - 2] + x[n - 4]).$

(b) $h[n] = \delta[n] + \delta[n - 1] + \delta[n - 2] + \delta[n - 3] + \delta[n - 4].$

(c) $H(e^{j\hat{\omega}}) = [1 + \cos(2\hat{\omega})]e^{-j\hat{\omega}3}.$

(d) $H(z) = 1 - 2z^{-2} + z^{-4} + z^{-7}.$



$$a) \quad h[n] = \delta[n] + 2\delta[n-2] + \delta[n-4]$$

(by definition of impulse response, $h[n]$ is the output generated by $x[n] = \delta[n]$)

$$\mathcal{H}(\hat{\omega}) = 1 + 2e^{-j2\hat{\omega}} + e^{-j4\hat{\omega}}$$

$$H(z) = 1 + 2z^{-2} + z^{-4}$$

$$b) \quad y[n] = x[n] + x[n-1] + x[n-2] + x[n-3] + x[n-4]$$

$$\mathcal{H}(\hat{\omega}) = 1 + e^{-j\hat{\omega}} + e^{-j2\hat{\omega}} + e^{-j3\hat{\omega}} + e^{-j4\hat{\omega}}$$

$$H(z) = 1 + z^{-1} + z^{-2} + z^{-3} + z^{-4}$$

$$c) \quad \mathcal{H}(\hat{\omega}) = \left(1 + \frac{e^{j2\hat{\omega}} + e^{-j2\hat{\omega}}}{2}\right) e^{-j3\hat{\omega}} =$$

$$= \frac{1}{2} e^{-j\hat{\omega}} + e^{-j3\hat{\omega}} + \frac{1}{2} e^{-j5\hat{\omega}}$$

$$H(z) = \frac{1}{2} z^{-1} + z^{-3} + \frac{1}{2} z^{-5}$$

$$h[n] = \frac{1}{2} \delta[n-1] + \delta[n-3] + \frac{1}{2} \delta[n-5]$$

$$y[n] = \frac{1}{2} x[n-1] + x[n-3] + \frac{1}{2} x[n-5]$$



$$d) \quad H(\hat{\omega}) = 1 - 2e^{-j2\hat{\omega}} + e^{-j4\hat{\omega}} + e^{-j7\hat{\omega}}$$

$$h[n] = \delta[n] - 2\delta[n-2] + \delta[n-4] + \delta[n-7]$$

$$y[n] = x[n] - 2x[n-2] + x[n-4] + x[n-7]$$