

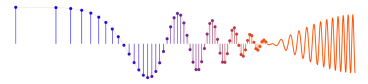


PROBLEM:

A linear time-invariant filter is described by the difference equation

$$y[n] = x[n] + x[n - 1] + x[n - 2] + x[n - 3] = \sum_{k=0}^3 x[n - k]$$

- What is the impulse response, $h[n]$, of this system?
- Determine the system function $H(z)$ for this system.
- Plot the zeros of $H(z)$ in the complex z -plane. *Hint: Remember the N -th roots of unity.*



$$\begin{aligned} \text{a) } h[n] &= \delta[n] + \delta[n-1] + \delta[n-2] + \delta[n-3] = \\ &= \sum_{k=0}^3 \delta[n-k] \end{aligned}$$

$$\begin{aligned} \text{b) } H(z) &= 1 + z^{-1} + z^{-2} + z^{-3} = \sum_{k=0}^3 z^{-k} = \frac{1 - z^{-4}}{1 - z^{-1}} = \\ &= \frac{z^4 - 1}{z^3(z-1)} \quad (\text{using the formula: } \sum_{k=0}^N x^k = \frac{1 - x^{N+1}}{1 - x}). \end{aligned}$$

$$\text{c) } H(z) = 0 \Rightarrow z^4 - 1 = 0 \Rightarrow z = \pm 1, \pm j$$

(the equation $z^N = 1$ has N solutions:

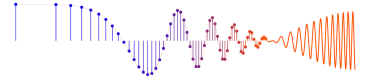
$$z = e^{j \frac{2k\pi}{N}}, \quad k = 0, 1, 2, \dots, N-1.$$

These values of z are called the N -th roots of unity).

However, the denominator of $\frac{z^4 - 1}{z^3(z-1)}$ also vanishes

for $z=1$, giving an indeterminate value $\frac{0}{0}$.

Using the expression $H(z) = \sum_{k=0}^3 z^{-k}$, we



obtain $H(1) = 4 \neq 0$. Therefore the zeros of $H(z)$ are $-1, \pm j$.

