PROBLEM:

A linear time-invariant filter is described by the difference equation

$$y[n] = x[n] + x[n-1] + x[n-2] + x[n-3] = \sum_{k=0}^{3} x[n-k]$$

(a) What is the impulse response, $h[n]$, of this system?

(b) Determine the system function $H(z)$ for this system.

(c) Plot the zeros of $H(z)$ in the complex z-plane. *Hint: Remember the N-th roots of unity.*

McClellan, Schafer and Yoder, *Signal Processing First*, ISBN 0-13-065562-7.
Prentice Hall, Upper Saddle River, NJ 07458.

SOLUTION

a) $h[n] = \delta[n] + \delta[n-1] + \delta[n-2] + \delta[n-3] =$

$$= \sum_{k=0}^{3} \delta[n-k]$$

b) $H(z) = 1 + z^{-1} + z^{-2} + z^{-3} = \sum_{k=0}^{3} z^{-k} = \dfrac{1 - z^{-4}}{1 - z^{-1}} =$

$$= \frac{z^4 - 1}{z^3(z-1)}$$ (using the formula: $\sum_{k=0}^{N} x^k = \dfrac{1 - x^{N+1}}{1 - x}$).

c) $H(z) = 0 \Rightarrow z^4 - 1 = 0 \Rightarrow z = \pm 1, \pm j$

(the equation $z^N = 1$ has N solutions:

$$z = e^{j\frac{2k\pi}{N}}, \quad k = 0, 1, 2, \dots, N-1.$$

These values of z are called the <u>N-th roots of unity</u>).

However, the denominator of $\dfrac{z^4 - 1}{z^3(z-1)}$ also vanishes for $z = 1$, giving an indeterminate value $\dfrac{0}{0}$.

Using the expression $H(z) = \sum_{k=0}^{3} z^{-k}$, we obtain $H(1) = 4 \neq 0$. Therefore the zeros of $H(z)$ are $-1, \pm j$.