PROBLEM:

The system function of a linear time-invariant filter is given by the formula

$$H(z) = (1 - z^{-1})(1 + z^{-2})(1 + z^{-1})$$

(a) Write the difference equation that gives the relation between the input $x[n]$ and the output $y[n]$.

(b) What is the output if the input is $x[n] = \delta[n]$?

(c) Use multiplication of z-transform polynomials to find the output when the input is

$$x[n] = 2\delta[n-1] + 2\delta[n-3] - 2\delta[n-4].$$

(d) If the input to the system is of the form

$$x[n] = e^{j\hat{\omega}n} \qquad -\infty < n < \infty,$$

for what values of $\hat{\omega}$ will the output be zero for all n? We cannot use z-transforms directly to solve this problem, but we can find the frequency response from $H(z)$ and then solve the problem. Note that the factored form will tell you the answer and so will the pole zero plot for $H(z)$.

McClellan, Schafer and Yoder, *Signal Processing First*, ISBN 0-13-065562-7.

SOLUTION

$$H(z) = (1-z^{-1})(1+z^{-1})(1+z^{-2}) = (1-z^{-2})(1+z^{-2}) =$$
$$= 1 - z^{-4}$$

a) $y[n] = x[n] - x[n-4]$

b) $h[n] = \delta[n] - \delta[n-4]$

c) $X(z) = 2z^{-1} + 2z^{-3} - 2z^{-4} = 2z^{-1}(1 + z^{-2} - z^{-3})$

$$H(z)X(z) = 2z^{-1}(1-z^{-4})(1+z^{-2}-z^{-3}) =$$
$$= 2z^{-1} + 2z^{-3} - 2z^{-4} - 2z^{-5} - 2z^{-7} + 2z^{-8} = Y(z)$$

$$y[n] = 2\delta[n-1] + 2\delta[n-3] - 2\delta[n-4] +$$
$$-2\delta[n-5] - 2\delta[n-7] + 2\delta[n-8]$$

d) From the factored form of $H(z)$, we see that the zeros of $H(z)$ are $z = 1, -1, j, -j$. These are all on the unit circle, and correspond to $\hat{\omega} = 0, \pi, \frac{\pi}{2}, -\frac{\pi}{2}$ respectively. For these values of $\hat{\omega}$ the output will be zero.

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