PROBLEM:

A linear time-invariant system has impulse response: $h(t) = e^{-(t-2)}u(t-2)$

(a) Plot $h(t-\tau)$ versus τ, for $t = 1$. Label your plot.

(b) Is the LTI system causal? Give a reason to support your answer.

(c) Is the system stable? Explain with a proof or counter-example.

(d) If the input is $x(t) = u(t)$, then it will be true that the output $y(t)$ is zero for $t \leq t_1$. Find t_1.

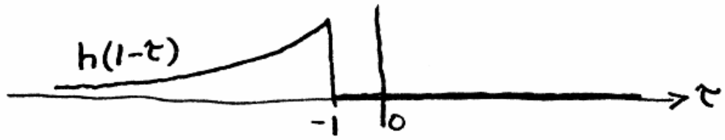
(e) The rest of the output signal (for $t > t_1$) is non-zero, when the input is $x(t) = u(t)$. Use the convolution integral to find the non-zero portion of the output, i.e., find $y(t)$ for $t > t_1$.
Hint: it might be easier to flip and slide $x(t)$.

McClellan, Schafer and Yoder, *Signal Processing First*, ISBN 0-13-065562-7.
Prentice Hall, Upper Saddle River, NJ 07458.

SOLUTION

(a) $h(t-\tau)$ for $t=1$ is $h(1-\tau) = e^{-(1-\tau-2)}u(1-\tau-2)$ ← FLIP & SHIFT by 1

$h(1-\tau) = e^{-(-1-\tau)}u(-1-\tau)$ ← STARTS @ $\tau=-1$



(b) Yes, the system is causal because $h(t)=0$ for $t<0$.
In fact, $h(t)=0$ for $t<2$.

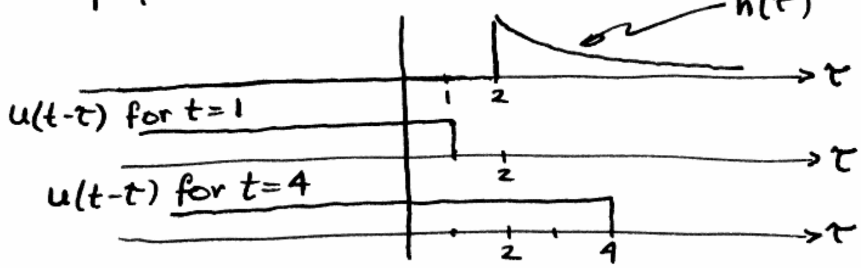
(c) To test for stability we do the integral $\int_{-\infty}^{\infty}|h(t)|dt$

$$\int_{-\infty}^{\infty}|e^{-(t-2)}u(t-2)|dt = \int_{2}^{\infty}e^{-(t-2)}dt = \left.\frac{e^{-(t-2)}}{-1}\right|_{2}^{\infty} = 0 - \frac{e^{0}}{-1} = 1 < \infty$$

Thus the system is stable.

(d) See the result from the convolution below: $t_1 = 2$

(e) Flip & Slide $x(t) = u(t)$



From the drawings, there is NO overlap when $t<2$.

$\Rightarrow y(t) = 0$ for $t<2$.

For $t \geq 2$, we have overlap from $\tau=2$ up to $\tau=t$.

$$y(t) = \int_{2}^{t} 1 \cdot e^{-(\tau-2)}d\tau = \left.\frac{e^{-(\tau-2)}}{-1}\right|_{2}^{t}$$

$$y(t) = \frac{e^{-(t-2)}}{-1} - \frac{e^{0}}{-1} = 1 - e^{-(t-2)}$$

$$\therefore y(t) = \left(1 - e^{-(t-2)}\right)u(t-2)$$