PROBLEM:

Determine the z-transforms of the following. *Express your answer as the ratio of polynomials in z^{-1}.*

(a) $x_a[n] = 2(0.8)^n u[n]$.

(b) $x_b[n] = 4(\frac{1}{2})^n u[n] - 2(\frac{1}{4})^{n-1} u[n-1]$.

(c) $x_b[n] = \delta[n] + u[n-1]$.

SOLUTION

a) $X_a[n] = 2(0.8)^n u[n]$, find $X_a(z)$

$$b_0 a_1^n u[n] \longrightarrow \frac{b_0}{1-a_1 z^{-1}} \qquad \text{from table + lecture}$$

$b_0 = 2$
$a_1 = 0.8$

$$\boxed{X_a(z) = \frac{2}{1-0.8 z^{-1}}}$$

b) $X_b[n] = 4(\frac{1}{2})^n u[n] - 2(\frac{1}{4})^{n-1} u[n-1]$, find $X_b(z)$

use linearity, transform from a),
and $x[n-1] \longrightarrow z^{-1} X(z)$

$$4(\tfrac{1}{2})^n u[n] \longrightarrow \frac{4}{1-0.5 z^{-1}}$$

$$2(\tfrac{1}{4})^n u[n] \longrightarrow \underset{\underset{X(z)}{''}}{\frac{2}{1-0.25 z^{-1}}}, \qquad x[n-1] \longrightarrow z^{-1}X(z) = \frac{2z^{-1}}{1-0.25 z^{-1}}$$

$$\underset{\underset{x[n]}{''}}{}$$

As polynomials in z^{-1}

$$\boxed{X_b(z) = \frac{4}{1-0.5 z^{-1}} - \frac{2 z^{-1}}{1-0.25 z^{-1}}} = \boxed{\frac{4 - 3z^{-1} + z^{-2}}{1-0.75 z^{-1} + 0.125 z^{-1}}}$$

c) $X_b[n] = \delta[n] + u[n-1]$, find $X_b(z)$

$$\delta[n] \longrightarrow 1 \quad, \qquad u[n] \longrightarrow \frac{1}{1-z^{-1}} , \quad u[n-1] \longrightarrow \frac{z^{-1}}{1-z^{-1}}$$

$$X_b(z) = 1 + \frac{z^{-1}}{1-z^{-1}} = \boxed{\frac{1}{1-z^{-1}}}$$

$$\left(\text{note, } X_b(z) \text{ corresponds to } X_b[n] = u[n], \right)$$
$$\text{but} \quad \delta[n] + u[n-1] = u[n]$$