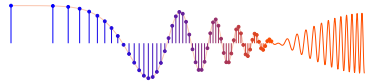

PROBLEM:

Determine the inverse z-transforms of the following:

(a) $H_a(z) = \dfrac{1 + z^{-2}}{1 - 0.8z^{-1}}.$

(b) $H_b(z) = \dfrac{0.5}{1 - 0.8e^{j0.25\pi}z^{-1}} + \dfrac{0.5}{1 - 0.8e^{-j0.25\pi}z^{-1}}.$

(c) $H_c(z) = \dfrac{1 + 0.64z^{-1}}{1 + 0.64z^{-2}}.$

McClellan, Schafer and Yoder, *Signal Processing First*, ISBN 0-13-065562-7.
Prentice Hall, Upper Saddle River, NJ 07458.

SOLUTION

a) $H_a(z) = \dfrac{1+z^{-2}}{1-0.8z^{-1}}$, find $h[n]$

$$\frac{1+z^{-2}}{1-0.8z^{-1}} = \frac{1}{1-0.8z^{-1}} + (z^{-2})\frac{1}{1-0.8z^{-1}}, \quad \frac{1}{1-0.8z^{-1}} \longrightarrow (0.8)^n u[n]$$

$$h[n] = (0.8)^n u[n] + (0.8)^{n-2} u[n-2]$$

b) $H_b(z) = \dfrac{0.5}{1-0.8e^{j0.25\pi}z^{-1}} + \dfrac{0.5}{1-0.8e^{-j0.25\pi}z^{-1}}$, find $h[n]$

recall $b_0 a_1^n u[n] \rightarrow \dfrac{b_0}{1-a_1 z^{-1}}$, ($a_1$ can be complex)
and use linearity

$$h_b[n] = (0.5)(0.8)^n (e^{j0.25\pi})^n u[n] + (0.5)(0.8)^n (e^{-j0.25\pi})^n u[n]$$

$$= \left(\frac{1}{2}\right)(0.8)^n u[n] \left[e^{j0.25\pi n} + e^{-j0.25\pi n}\right]$$

$$= (0.8)^n u[n] \cos(0.25\pi n)$$

(c) $H(z)=\dfrac{1+0.64z^{-1}}{1+0.64z^{-2}}$, find h[n].

$$\frac{1+0.64z^{-1}}{1+0.64z^{-2}}=\frac{1+0.64z^{-1}}{(1+j0.8z^{-1})(1-j0.8z^{-1})}=\frac{A}{1+j0.8z^{-1}}+\frac{B}{1-j0.8z^{-1}}$$

$$A=\left.\frac{1+0.64z^{-1}}{1-j0.8z^{-1}}\right|_{z=-j0.8}=\frac{1+\frac{0.64}{-j0.8}}{1-\frac{j0.8}{-j0.8}}=0.5+j0.4\approx 0.6403e^{j0.6747}$$

$$B=\left.\frac{1+0.64z^{-1}}{1+j0.8z^{-1}}\right|_{z=j0.8}=\frac{1+\frac{0.64}{j0.8}}{1+\frac{j0.8}{j0.8}}=0.5-j0.4\approx 0.6403e^{-j0.6747}$$

$$h[n]=(0.5+j0.4)(-j0.8)^n u[n]+(0.5-j0.4)(j0.8)^n u[n]$$

$$=(0.5+j0.4)\left(0.8e^{-j\frac{\pi}{2}}\right)^n u[n]+(0.5-j0.4)\left(0.8e^{j\frac{\pi}{2}}\right)^n u[n]$$

$$=(0.8)^n u[n]\left(0.5\left(\left(e^{-j\frac{\pi}{2}}\right)^n+\left(e^{j\frac{\pi}{2}}\right)^n\right)+j0.4\left(\left(e^{-j\frac{\pi}{2}}\right)^n-\left(e^{j\frac{\pi}{2}}\right)^n\right)\right)$$

$$=(0.8)^n u[n]\left(\cos\left(\frac{\pi}{2}n\right)+j0.4\left(-j2\sin\left(\frac{\pi}{2}n\right)\right)\right)$$

$$=(0.8)^n u[n]\left(\cos\left(\frac{\pi}{2}n\right)+0.8\sin\left(\frac{\pi}{2}n\right)\right)$$

$$=(0.8)^n u[n]\left(\cos\left(\frac{\pi}{2}n\right)+0.8\cos\left(\frac{\pi}{2}n-\frac{\pi}{2}\right)\right)$$

$$=(0.8)^n u[n]\left(\Re\left\{e^{j\frac{\pi}{2}n}+0.8e^{j\frac{\pi}{2}n}e^{-j\frac{\pi}{2}}\right\}\right)$$

$$=(0.8)^n u[n]\left(\Re\left\{e^{j\frac{\pi}{2}n}\left(1+0.8e^{-j\frac{\pi}{2}}\right)\right\}\right)$$

$$=(0.8)^n u[n]\left(\Re\left\{e^{j\frac{\pi}{2}n}(1-j0.8)\right\}\right)$$

$$\approx(0.8)^n u[n]\left(\Re\left\{e^{j\frac{\pi}{2}n}\left(1.2806e^{-j0.6747}\right)\right\}\right)$$

$$=\left(1.2806\cos\left(\frac{\pi}{2}n-0.6747\right)\right)(0.8)^n u[n]$$