PROBLEM:

For each of the difference equations below, determine the poles and zeros of the corresponding system function, $H(z)$. Plot the poles **(X)** and zeros **(O)** in the complex z-plane.

$$\mathcal{S}_1: \qquad y[n] = 0.8y[n-1] + x[n] + x[n-2]$$

$$\mathcal{S}_2: \qquad y[n] = 0.8y[n-1] + 1.25x[n] - x[n-1]$$

$$\mathcal{S}_3: \qquad y[n] = -0.64y[n-2] + x[n] + 0.64x[n-1]$$

$$\mathcal{S}_4: \qquad y[n] = x[n] + \tfrac{3}{4}x[n-1] - \tfrac{1}{4}x[n-2]$$

McClellan, Schafer and Yoder, *Signal Processing First*, ISBN 0-13-065562-7.

SOLUTION

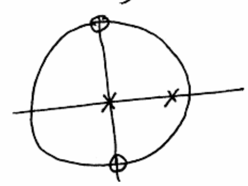
find poles and zeros

S_1: $Y(z) = 0.8 z^{-1} Y(z) + X(z) + z^{-2} X(z)$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1 + z^{-2}}{1 - 0.8 z^{-1}} \text{, multiply by } \frac{z^2}{z^2} \Rightarrow \frac{z^2 + 1}{(z)(z - 0.8)}$$

poles @ $(z)(z-0.8) = 0 \quad \Rightarrow \quad z = 0,\ 0.8$

zeros @ $z^2 + 1 = 0$, $z = \pm j$

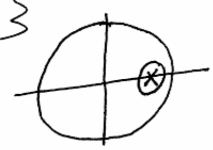


S_2: $H(z) = \dfrac{1.25 - z^{-1}}{1 - 0.8 z^{-1}} = \dfrac{1.25 z - 1}{z - 0.8}$

zero @ $z = 0.8$

pole @ $z = 0.8$

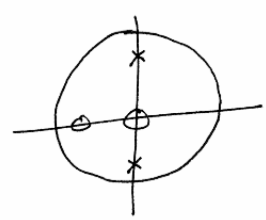
$\left\{ \text{note } H(z) = (1.25)\left(\dfrac{1.25 - z^{-1}}{1.25 - z^{-1}}\right) = 1.25 \right\}$



S_3: $H(z) = \dfrac{1 + 0.64 z^{-1}}{1 + 0.64 z^{-2}} = \dfrac{z^2 + 0.64 z}{z^2 + 0.64} = \dfrac{(z)(z + 0.64)}{(z + 0.8j)(z - 0.8j)}$

zeros @ $z = 0,\ -0.64$

poles @ $z = \pm 0.8j$



S_4: $H(z) = \dfrac{1 + \frac{3}{4} z^{-1} - \frac{1}{4} z^{-2}}{1} = \dfrac{z^2 + \frac{3}{4} z - \frac{1}{4}}{z^2} = \dfrac{(z+1)(z - \frac{1}{4})}{(z)(z)}$

zeros @ $z = -1$, $z = \frac{1}{4}$

poles @ $z = 0, 0$