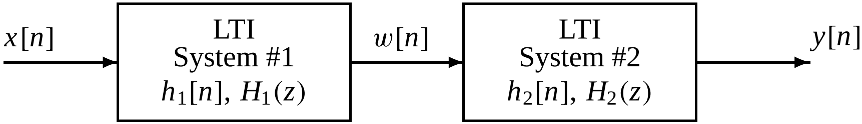
PROBLEM:

A cascade of two discrete-time systems is depicted by the following block diagram:



System #1 is defined by the system function $H_1(z) = z^{-1}(1 - 0.5z^{-1})$ and System #2 is defined by the difference equation $y[n] = 0.8y[n-1] + w[n]$.

(a) If the input to the first system is $x[n] = \delta[n] + \delta[n-1]$, determine the output, $w[n]$, of the **first** system.

$w[n] =$

(b) Determine the system function $H(z)$ of the overall system.

$H(z) =$

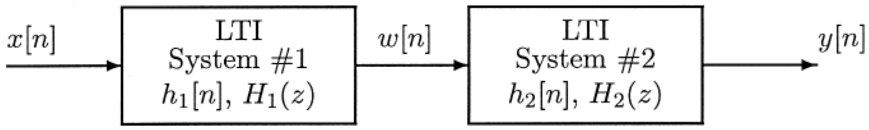
(c) Determine the impulse response of the overall system.

$h[n] =$

McClellan, Schafer and Yoder, *Signal Processing First*, ISBN 0-13-065562-7.

SOLUTION

A cascade of two discrete-time systems is depicted by the following block diagram:



System #1 is defined by the system function $H_1(z) = z^{-1}(1 - 0.5z^{-1})$ and System #2 is defined by the difference equation $y[n] = 0.8y[n-1] + w[n]$.

(a) If the input to the first system is $x[n] = \delta[n] + \delta[n-1]$, determine the output, $w[n]$, of the **first** system.

$$W(z) = H_1(z)X(z)$$
$$= z^{-1}(1 - \tfrac{1}{2}z^{-1})(1 + z^{-1})$$
$$= z^{-1} + \tfrac{1}{2}z^{-2} - \tfrac{1}{2}z^{-3}$$

invert

$$\delta[n-1] \qquad \tfrac{1}{2}\delta[n-2] \qquad -\tfrac{1}{2}\delta[n-3]$$

$$\boxed{w[n] = \delta[n-1] + \tfrac{1}{2}\delta[n-2] - \tfrac{1}{2}\delta[n-3]}$$

(b) Determine the system function $H(z)$ of the overall system.

$$H(z) = H_1(z)H_2(z)$$
$$= z^{-1}(1 - \tfrac{1}{2}z^{-1})\left(\frac{1}{1 - 0.8z^{-1}}\right)$$

$$\boxed{H(z) = \frac{z^{-1} - \tfrac{1}{2}z^{-2}}{1 - 0.8z^{-1}}}$$

(c) Determine the impulse response of the overall system.

$$\frac{z^{-1}}{1 - 0.8z^{-1}} \xrightarrow[z]{\text{inverse}} (0.8)^{n-1}u[n-1]$$

$$\frac{-\tfrac{1}{2}z^{-2}}{1 - 0.8z^{-1}} \longrightarrow -\tfrac{1}{2}(0.8)^{n-2}u[n-2]$$

$$\boxed{h[n] = (0.8)^{n-1}u[n-1] - \tfrac{1}{2}(0.8)^{n-2}u[n-2]}$$