

PROBLEM:

A cascade of two discrete-time systems is depicted by the following block diagram:

$$x[n] \xrightarrow{\text{LTI}} w[n] \xrightarrow{\text{LTI}} y[n]$$

$$x[n] \xrightarrow{\text{LTI}} y[n]$$

$$h_1[n], H_1(z) \xrightarrow{\text{LTI}} h_2(z)$$

System #1 is defined by the system function $H_1(z) = z^{-1}(1 - 0.5z^{-1})$ and System #2 is defined by the difference equation y[n] = 0.8y[n - 1] + w[n].

(a) If the input to the first system is $x[n] = \delta[n] + \delta[n-1]$, determine the output, w[n], of the **first** system.

w[n] =

(b) Determine the system function H(z) of the overall system.

H(z) =

(c) Determine the impulse response of the overall system.

h[n] =

McClellan, Schafer and Yoder, Signal Processing First, ISBN 0-13-065562-7. Prentice Hall, Upper Saddle River, NJ 07458. © 2003 Pearson Education, Inc.





A cascade of two discrete-time systems is depicted by the following block diagram:

$$\begin{array}{c|c} x[n] & & LTI \\ System \#1 \\ h_1[n], H_1(z) & & & & \\ \end{array} \begin{array}{c} w[n] & & LTI \\ System \#2 \\ h_2[n], H_2(z) & & & \\ \end{array} \end{array} \begin{array}{c} y[n] \\ \end{array}$$

System #1 is defined by the system function $H_1(z) = z^{-1}(\overline{1-0.5z^{-1}})$ and System #2 is defined by the difference equation y[n] = 0.8y[n-1] + w[n].

(a) If the input to the first system is $x[n] = \delta[n] + \delta[n-1]$, determine the output, w[n], of the first system.

$$W(z) = H_{1}(z) X(z)$$

$$= z^{-1} (1 - \frac{1}{2} z^{-1}) (1 + z^{-1})$$

$$= z^{-1} + \frac{1}{2} z^{-2} - \frac{1}{2} z^{-3}$$
invert
$$\int \int \int z^{-1} z^{-1} z^{-3} - \frac{1}{2} \delta[n-3]$$

$$w[n] = \delta[n-1] + \frac{1}{2} \delta[n-2] - \frac{1}{2} \delta[n-3]$$

(b) Determine the system function H(z) of the overall system.

$$H(z) = H_{1}(z) H_{2}(z)$$

= $z^{-1}(1 - \frac{1}{2}z^{-1}) \left(\frac{1}{1 - 0.8z^{-1}}\right)$

$$H(z) = \frac{z^{-1} - \frac{1}{2} z^{-2}}{1 - 0.8 z^{-1}}$$

(c) Determine the impulse response of the overall system.

$$\frac{z^{-1}}{1-0.8z^{-1}} \xrightarrow{\text{inverse}} (0,8)^{n-1} u[n-1]$$

$$\frac{-\frac{1}{2}z^{-2}}{1-0.8z^{-1}} \xrightarrow{-\frac{1}{2}} (0.8)^{n-2} u[n-2]$$

$$h[n] = (0.8)^{n-1} u[n-1] - \frac{1}{2} (0.8)^{n-2} u[n-2]$$

McClellan, Schafer, and Yoder, Signal Processing First, ISBN 0-13-065562-7. Prentice Hall, Upper Saddle River, NJ 07458. © 2003 Pearson Education, Inc.