PROBLEM:

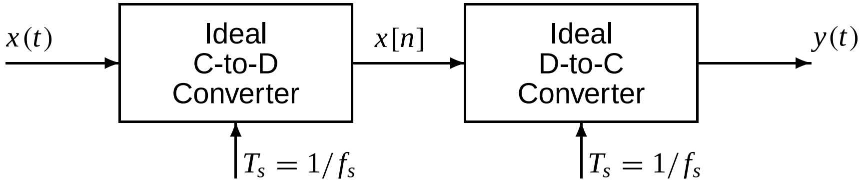


Figure 1: Ideal sampling and reconstruction system.

Shown in the figure above is an ideal C-to-D converter that samples $x(t)$ with a sampling period $T_s = 1/f_s$ to produce the discrete-time signal $x[n]$. The ideal D-to-C converter then forms a continuous-time signal $y(t)$ from the samples $x[n]$. Suppose that $x(t)$ is given by

$$x(t) = [10 + 10\cos(500\pi t - \pi/2)]\cos(2000\pi t)$$

(a) Use Euler's formulas for the cosine functions to expand $x(t)$ in terms of complex exponential signals so that you can sketch the two-sided spectrum of this signal. Be sure to label important features of the plot. Is this waveform periodic? If so, what is the period?

(b) What is the *minimum* sampling rate f_s that can be used in the above system so that $y(t) = x(t)$?

(c) Plot the spectrum of the sampled signal $x[n]$ for the case when $f_s = 5000$. Your plot should include labels on the frequency (on the $\hat{\omega}$ scale), amplitude and phase of each spectrum component.

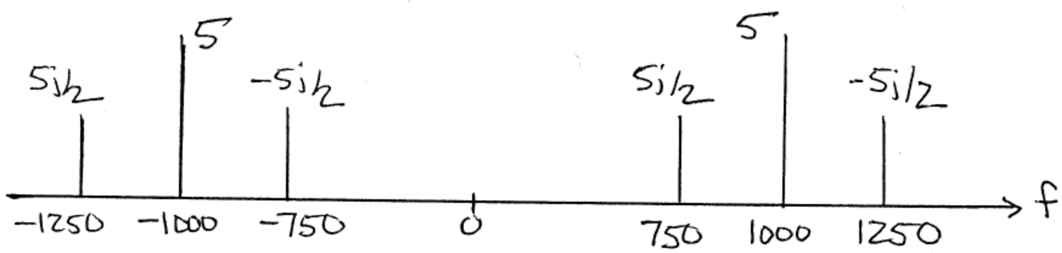
McClellan, Schafer and Yoder, *Signal Processing First*, ISBN 0-13-065562-7.

SOLUTION

(a) $x(t) = \left[10 + 10\cos(500\pi t - \pi/2)\right]\cos(2000\pi t)$

$$= \left[10 + 5e^{-j\pi/2}e^{j500\pi t} + 5e^{j\pi/2}e^{-j500\pi t}\right] \cdot \left[\tfrac{1}{2}e^{j2000\pi t} + \tfrac{1}{2}e^{-j2000\pi t}\right]$$

$$= 5e^{j2000\pi t} + 5e^{-j2000\pi t} - \frac{5j}{2}e^{j2500\pi t} - \frac{5j}{2}e^{-j1500\pi t} + \frac{5j}{2}e^{j1500\pi t} + \frac{5j}{2}e^{-j2500\pi t}$$



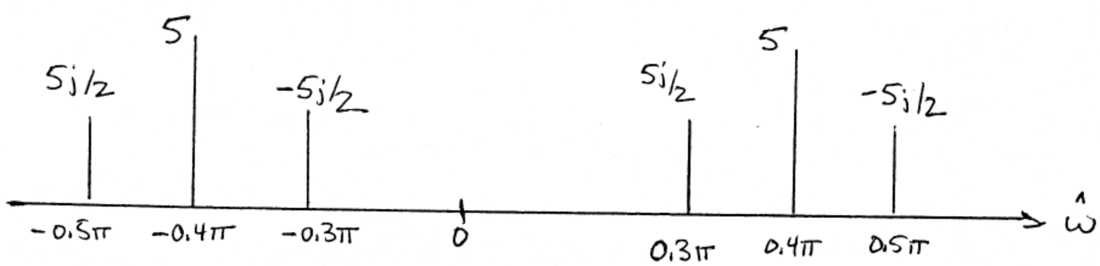
ALL FREQUENCIES ARE MULTIPLES OF 250 Hz.

$\Rightarrow x(t)$ IS PERIODIC WITH PERIOD $T_0 = \frac{1}{250}$ s.

(b) $f_s > 2(1250) = 2500$ SAMPLES/SEC.

(c) $x[n] = x\left(\frac{n}{5000}\right) = 5e^{j0.4\pi n} + 5e^{-j0.4\pi n}$

$$-\frac{5j}{2}e^{j0.5\pi n} - \frac{5j}{2}e^{-j0.3\pi n} + \frac{5j}{2}e^{j0.3\pi n} + \frac{5j}{2}e^{j0.5\pi n}$$



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