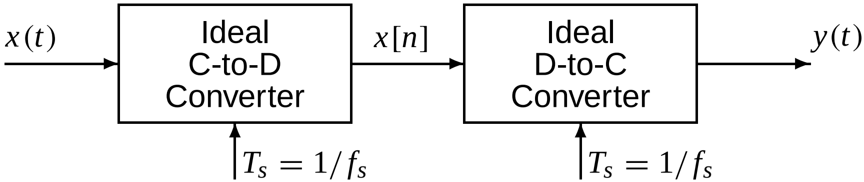
PROBLEM:

Again consider the ideal sampling and reconstruction system

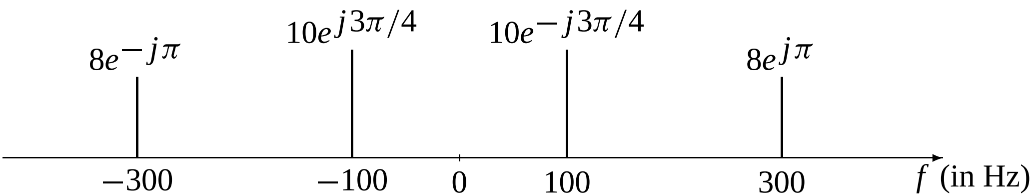


(a) Suppose that the discrete-time signal $x[n]$ in the figure above is given by the formula

$$x[n] = 4\cos(0.3\pi n - \pi/3)$$

If the sampling rate of the C-to-D converter is $f_s = 10000$ samples/second, many *different* continuous-time signals $x(t) = x_\ell(t)$ could have been inputs to the above system. Determine two such inputs with frequency less than 10000 Hz; i.e., find $x_1(t) = A_1\cos(\omega_1 t + \phi_1)$ and $x_2(t) = A_2\cos(\omega_2 t + \phi_2)$ such that $x[n] = x_1(nT_s) = x_2(nT_s)$ if $T_s = 1/10000$ secs.

(b) Now if the input $x(t)$ to the system in the figure above has the two-sided spectrum representation shown below,what is the *minimum* sampling rate f_s such that the output $y(t)$ is equal to the input $x(t)$?



(c) Determine the spectrum for $x[n]$ when $f_s = 300$ samples/sec. Make a $\boxed{\text{plot}}$ for your answer, but label the frequency, amplitude and phase of each spectral component.

McClellan, Schafer and Yoder, *Signal Processing First*, ISBN 0-13-065562-7.

SOLUTION

(a) $x[n] = x(nT) = 4\cos(0.3\pi n - \pi/3)$ for $T = \frac{1}{10{,}000}$

IF NO ALIASING: $x_1(t) = 4\cos\left(0.3(10{,}000)\pi t - \pi/3\right)$

$$= 4\cos(3000\pi t - \pi/3)$$

IF FOLDING: $x_2(t) = 4\cos\left((0.3\pi - 2\pi)10{,}000\, t - \pi/3\right)$

$$= 4\cos(-17{,}000\, t - \pi/3)$$

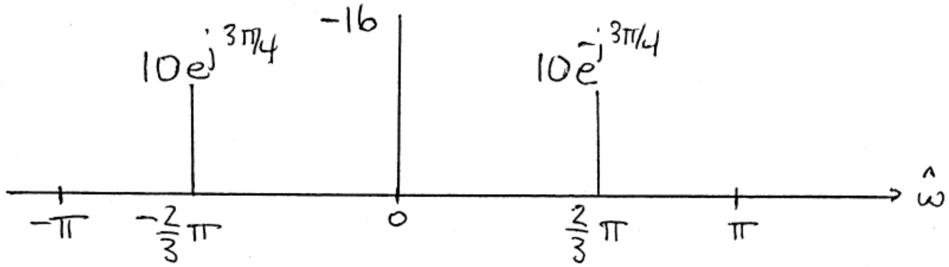
$$= 4\cos(17{,}000\, t + \pi/3)$$

(b) $f_s > 2(300) = 600$ samples/sec.

(c) $x(t) = 20\cos(200\pi t - 3\pi/4) + 16\cos(600\pi t + \pi)$

$$x[n] = x(t)\Big|_{t=\frac{n}{300}} = 20\cos\left(\tfrac{2}{3}\pi n - 3\pi/4\right) + 16\cos(2\pi n + \pi)$$

$$= 20\cos\left(\tfrac{2}{3}\pi n - \tfrac{3\pi}{4}\right) - 16$$



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