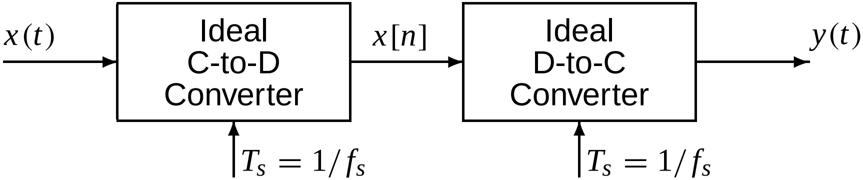
**PROBLEM:**




We can do some interesting things with sampling. One of them is that we can change the period of a periodic waveform. This problem illustrates how this can be done for the specific periodic input signal

$$x(t) = 2\cos(2\pi(33)t) + \cos(2\pi(99)t).$$

In all the following parts, assume that the sampling frequency is $f_s = 30$ Hz. Note that this sampling rate *does not* satisfy the conditions of the Shannon sampling theorem, so aliasing will occur.

(a) Plot the spectrum of the periodic continuous-time signal $x(t)$. What is the fundamental frequency of $x(t)$?

(b) Determine an expression for the discrete-time signal $x[n]$ as a sum of discrete-time cosine signals. Be sure that all of the normalized frequencies are positive and less than $\pi$ radians. Plot the spectrum of $x[n]$ over the range of normalized frequencies $-\pi \le \hat{\omega} \le \pi$.

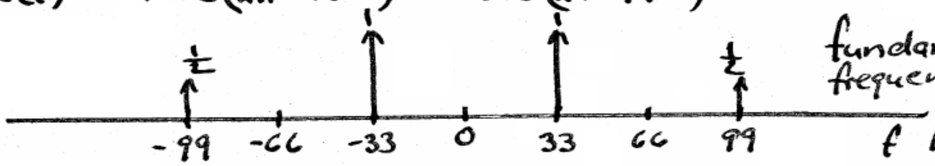
(c) Now the continuous-time output signal $y(t)$ that is created by the ideal D-to-C converter operating with sampling rate $f_s = 30$ Hz will also be a sum of cosine signals. Write an expression for $y(t)$ and plot its spectrum. What is the fundamental frequency of $y(t)$?

(d) How are the fundamental frequencies of $x(t)$ and $y(t)$ related? Do you think that it would be possible to change the fundamental frequency by a different factor by using a different sampling rate?

McClellan, Schafer and Yoder, *Signal Processing First*, ISBN 0-13-065562-7.

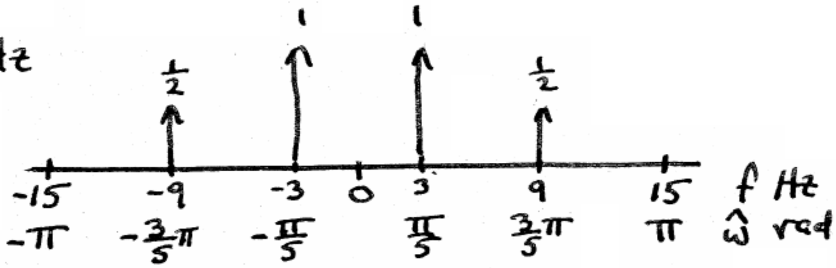

**SOLUTION**

a) $x(t) = 2\cos(2\pi \cdot 33t) + \cos(2\pi \cdot 99t)$




fundamental frequency 33 Hz

b) $f_s = 30$ Hz

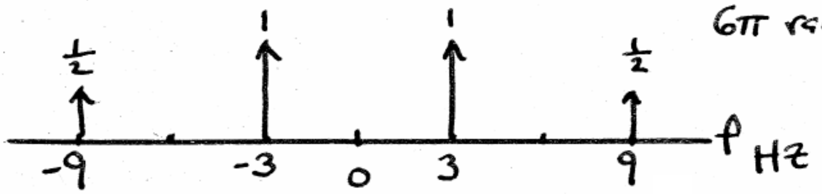



$x[n] = 2\cos\frac{\pi}{5}n + \cos\frac{3}{5}\pi n$

c) $y(t) \overset{f_s = 30\text{ Hz}}{=} 2\cos\frac{\pi}{5}\cdot 30t + \cos\frac{3}{5}\pi\cdot 30t$

$= 2\cos 6\pi t + \cos 18\pi t$

fundamental frequency $6\pi$ rad/sec or 3 Hz




d) We observe for $f_s = 30$ Hz, $x(t)$ has a fundamental frequency of 33 Hz and $y(t)$ has a fundamental frequency of 3 Hz — an 11 fold difference in frequency this illustrates that the fundamental frequency can be changed by altering the sampling rate of a sampling system.

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