PROBLEM:

Consider a continuous-time signal

$$x(t) = A\cos(\omega_0 t + \phi)$$

We know that this signal is periodic with period $T_0 = 2\pi/\omega_0$; i.e. $x(t + T_0) = x(t)$ for all t. Now suppose that $x(t)$ is sampled to obtain the sequence

$$x[n] = x(nT_s) = A\cos(\omega_0 nT_s + \phi) = A\cos(\hat{\omega}_0 n + \phi)$$

where $\hat{\omega}_0 = \omega_0 T_s$.

Now a discrete-time signal is periodic with period N if $x[n+N] = x[n]$ for all n, where N is necessarily an integer.

(a) Will $x[n]$ be periodic for all possible sampling rates? If not, what condition on T_s will ensure that $x[n]$ is periodic with period N?

(b) If $\omega_0 = 2000\pi$, what value of T_s will result in a periodic sequence with period $N = 100$?

McClellan, Schafer and Yoder, *Signal Processing First*, ISBN 0-13-065562-7.
Prentice Hall, Upper Saddle River, NJ 07458.

SOLUTION

a) $x(t) = A\cos(\omega_0 t + \phi)$

$x(n) = x(nT_s) = A\cos(\hat{\omega}_0 n + \phi)$ where $\hat{\omega}_0 = \omega_0 T_s$

periodicity of $x(n)$ implies

$$\hat{\omega}_0(n+N) = \hat{\omega}_0 n + 2\pi k$$ so that

$$A\cos(\hat{\omega}_0(n+N)+\phi) = A\cos(\hat{\omega}_0 n + \phi)$$

and $\therefore$ $x[n] = x[n+N]$

Thus $\hat{\omega}_0 N$ must be a multiple of 2π

$$\hat{\omega}_0 N = 2\pi k \qquad k = 1, 2, \ldots$$

$\therefore$ $\omega_0 T_s N = 2\pi k$ For periodicity, T_s must be chosen so that

$$T_s = \frac{2\pi k}{\omega_0 N} \qquad k = 1, 2, 3, \ldots$$

b) If $\omega_0 = 2000\pi$ and $N = 100$

$$T_s = \frac{2\pi k}{2000\pi \cdot 100} = 10^{-5} k \qquad k = 1, 2, 3 \ldots$$

The fundamental period corresponds to $k = 1$

$\therefore$ $T_s = 10^{-5}$ seconds

McClellan, Schafer, and Yoder, *Signal Processing First*, ISBN 0-13-065562-7.