PROBLEM:

The following is a simple problem that might be posed about an LTI system:

> *Given the input sequence* $x[n]$ *find the output sequence* $y[n]$ *of a 5-point running average filter for all values of* n.

The following is a partial list of possible approaches to solving this problem:

1. Use the difference equation representation of the system to compute (e.g., using MATLAB) the output $y[n]$ for all required values of n.

2. Multiply the z-transform of the input by the system function and determine $y[n]$ as the inverse z-transform of $Y(z)$.

3. Break the input into a sum of complex exponential signals, use the frequency response function to determine the output due to each complex exponential signal separately, and finally, add the individual outputs together to get $y[n]$.

4. Some combination of the above methods. Remember, when the input is a sum of two or more signals and the system is linear, we can solve the problem separately for each of the input components and then superimpose the outputs. We can therefore use the method that is most appropriate for each of the components of the input.

In each of these solution methods you would use one or more of the basic representations of the 5-point running average filter. Which method is easiest will have a lot to do with the nature of the input signal. This may require that you convert a given representation of the system into one of the other forms. For example, if you are given the difference equation and you want to use approach #2, you will have to determine the system function $H(z)$ from the difference equation coefficients.

Now in each of the following cases, the input will be given. In each case, determine which representation of the system and which of the above approaches will lead to the easiest solution of the problem, and detail the steps in using that approach to solve the problem. For example, if you choose approach #2 to solve the problem, your answer should be something like the following:

Step 1 Find $X(z)$, the z-transform of $x[n]$.

Step 2 Find $H(z)$, the system function of the 5-point running averager.

Step 3 Multiply $X(z)H(z)$ to get $Y(z)$.

Step 4 Find the inverse z-transform of $Y(z)$ to get $y[n]$.

Now here are some possible inputs. In each case, state which of the above (#1, #2, or #3) approaches you would use. There may not be a clear cut answer. Give the approach that you *think* will yield the solution with least effort. Outline your approach to solving the problem of finding the output of the 5-point moving averager. **You do not have to actually find the output—just tell how you would solve it in a step-by-step procedure described as illustrated above.**

(a) $x[n]$ is a sampled audio signal. It is represented by a vector of 100000 numbers.

(b) $x[n] = 4\cos(0.1\pi n + \pi/2) + 3\cos(0.4\pi n - \pi)$ for $-\infty < n < \infty$.

(c) $x[n] = \begin{cases} 1 & 0 \le n \le 10 \\ 0 & \text{otherwise.} \end{cases}$

(d) $x[n] = 10\delta[n-50]$.

(e) $x[n] = 10\delta[n-50] + 4\cos(0.1\pi n + \pi/2) + 3\cos(0.4\pi n - \pi)$ for $-\infty < n < \infty$.

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SOLUTION

5-point running average = LTI system

System representations:

(1) Difference equation

$$y(n) = \frac{1}{5}\sum_{k=0}^{4} x(n-k)$$

(2) Impulse response

$$h(n) = \frac{1}{5}\delta(n) + \frac{1}{5}\delta(n-1) + \frac{1}{5}\delta(n-2) + \frac{1}{5}\delta(n-3) + \frac{1}{5}\delta(n-4)$$

(3) Frequency response

$$\mathcal{H}(\hat{\omega}) = \frac{1}{5}\left[1 + e^{-j\hat{\omega}} + e^{-j2\hat{\omega}} + e^{-j3\hat{\omega}} + e^{-j4\hat{\omega}}\right]$$

(4) System function

$$H(z) = \frac{1}{5}\left(1 + z^{-1} + z^{-2} + z^{-3} + z^{-4}\right)$$

(a)

$x(n)$ = sampled audio signal represented by 100000 numbers

Step 1: Compute the difference equation that describes the LTI system. (representation (1))

Step 2: Use the difference equation (with Matlab) to calculate $y(n)$

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(b)

$x(n) = 4\cos(0.1\pi n + \pi/2) + 3\cos(0.4\pi n - \pi) \quad -\infty < n < +\infty$

Step 1 : Find the frequency response $\mathcal{H}(\hat{\omega})$ of the LTI system [representation (3)]

Step 2 : Evaluate $\mathcal{H}(\hat{\omega} = 0.1\pi)$ and $\mathcal{H}(\hat{\omega} = 0.4\pi)$

Step 3 : Obtain response $y_1(n)$ to $4\cos(0.1\pi n + \pi/2)$ by multiplying $|\mathcal{H}(\hat{\omega}=0.1\pi)|\, 4\cos(0.1\pi n + \frac{\pi}{2} + \angle\mathcal{H}(\hat{\omega}=0.1\pi))$

Step 4 : Obtain response $y_2(n)$ to $3\cos(0.4\pi n - \pi)$ by multiplying $|\mathcal{H}(\hat{\omega}=0.4\pi)|\, 3\cos(0.4\pi n - \pi + \angle\mathcal{H}(\hat{\omega}=0.4\pi))$

Step 5 : Using linearity property of the system find $y(n)$ by adding $y_1(n)$ and $y_2(n)$

(c)

$$x(n) = \begin{cases} 1 & 0 \le n \le 10 \\ 0 & \text{otherwise} \end{cases}$$

Step 1 : Find the impulse response $h(n)$ [representation (2)]

Step 2 : Use convolution to find $y(n) = h(n) * x(n)$

(d)

$x(n) = 10\,\delta(n-50)$

Step 1 : Find the impulse response $h(n)$ [representation (2)]

Step 2 : Use convolution to find $y(n) = h(n) * x(n) \Rightarrow$

$$y(n) = h(n) * 10\,\delta(n-50) = 10\,h(n-50)$$
$$= 2\sum_{\ell=0}^{4} \delta(n-\ell-50)$$

useful property: $h(n) * \delta(n) = h(n)$

$h(n) * \delta(n-n_0) = h(n-n_0)$

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(e) $x(n) = 10\,\delta(n-50) + 4\cos(0.1\pi n + \pi/2) + 3\cos(0.4\pi n - \pi)$

$-\infty < n < +\infty$

This requires a combination of approaches

Step 1 : Find impulse response $h(n)$ [representation (2)]

Step 2 : Use convolution to find $y_1(n)$ to input $10\delta(n-50)$

Step 3: Find frequency response $\mathcal{H}(\hat{\omega})$ [representation (3)]

Step 4 : Obtain response $y_2(n)$ to input $4\cos(0.1\pi n + \pi/2)$

$$y_2(n) = |\mathcal{H}(\hat{\omega} = 0.1\pi)|\, 4\cos(0.1\pi n + \tfrac{\pi}{2} + \angle\mathcal{H}(\hat{\omega}=0.1\pi))$$

Step 5 : Obtain response $y_3(n)$ to input $3\cos(0.4\pi n - \pi)$

$$y_3(n) = |\mathcal{H}(\hat{\omega} = 0.4\pi)|\, 3\cos(0.4\pi n - \pi + \angle\mathcal{H}(\hat{\omega}=0.4\pi))$$

Step 6 : Using the linearity of the system find

$$y(n) = y_1(n) + y_2(n) + y_3(n)$$