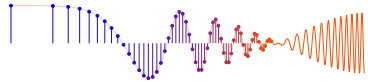


PROBLEM:

Consider the following MATLAB program:

```
nn = 0:16000;
xx = 3 + 2*cos(0.75*pi*nn-pi/4) + 11*cos(1.5*pi*nn-pi/3);
yy = conv([1,0,0,0,-1]/4,xx);
soundsc(yy,8000)
```

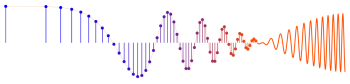
(a) What is the system function $H(z)$ of the system that is implemented by the `conv( )` statement?

(b) What is the frequency response of the system?

(c) Neglecting the end effects in the convolution, determine $y(t)$ that describes the signal produced by the `soundsc( )` statement.

McClellan, Schafer and Yoder, *Signal Processing First*, ISBN 0-13-065562-7.
Prentice Hall, Upper Saddle River, NJ 07458.

SOLUTION



(a) $h(n) = \frac{1}{4}[\delta(n) - \delta(n-4)]$

$H(z) = \sum_n h(n) z^{-n} = \frac{1}{4}[1 - z^{-4}]$

(b) $\mathcal{H}(\hat{\omega}) = H(z)\Big|_{z=e^{j\hat{\omega}}} = \frac{1}{4}[1 - e^{-j4\hat{\omega}}]$

(c) $x(n) = 3 + 2\cos(0.75\pi n - \pi/4) + 11\cos(\underbrace{1.5\pi n}_{\text{aliasing}} - \pi/3)$

$= 3 + 2\cos(0.75\pi n - \pi/4) + 11\cos((1.5\pi - 2\pi)n - \pi/3) =$

$= 3 + 2\cos(0.75\pi n - \pi/4) + 11\cos(-0.5\pi n - \pi/3) =$

$= 3 + 2\cos(0.75\pi n - \pi/4) + 11\cos(0.5\pi n + \pi/3)$

$$y(n) = 3\,\mathcal{H}(\hat{\omega}=0) + 2\,|\mathcal{H}(w=0.75\pi)|\cos(0.75\pi n - \tfrac{\pi}{4} + \underline{\angle\mathcal{H}(\hat{\omega}=0.75\pi)}) + 11\,|\mathcal{H}(w=0.5\pi)|\cos(0.5\pi n + \pi/3 + \underline{\angle\mathcal{H}(\hat{\omega}=0.5\pi)})$$

$\mathcal{H}(\hat{\omega}=0) = \frac{1}{4}(1 - e^{-j4\cdot 0}) = \frac{1}{4}(1-1) = 0$

$\mathcal{H}(\hat{\omega}=0.5\pi) = \frac{1}{4}(1 - e^{-j4\cdot 0.5\pi}) = \frac{1}{4}(1 - e^{-j2\pi}) = \frac{1}{4}(1-1) = 0$

$\mathcal{H}(\hat{\omega}=0.75\pi) = \frac{1}{4}(1 - e^{-j3\pi}) = \frac{1}{4}(1 - e^{-j\pi}) = \frac{1}{4}(1-(-1)) = \frac{1}{2}$

Therefore,

$$y(n) = 2\cdot\frac{1}{2}\cos(0.75\pi n - \pi/4 + \phi) = \cos(0.75\pi n - \pi/4)$$

Then $y(t) = y(n)\Big|_{n=tf_s} =$

$= \cos(0.75\pi \cdot t \cdot 8000 - \pi/4) =$

$= \cos(6000\pi t - \pi/4)$