PROBLEM:

We now have four ways of describing an LTI system: the difference equation; the impulse response, $h[n]$; the frequency response, $H(e^{j\hat{\omega}})$; and the system function, $H(z)$. In the following, you are given one of these representations and you must find the other three.

(a) $y[n] = (x[n] + 2x[n-2] + x[n-4])$.

(b) $h[n] = \delta[n] + \delta[n-1] + \delta[n-2] + \delta[n-3] + \delta[n-4]$.

(c) $H(e^{j\hat{\omega}}) = [1 + \cos(2\hat{\omega})]e^{-j\hat{\omega}3}$. *Hint: Expand the cosine using Euler's formula.*

(d) $H(z) = 1 - 2z^{-2} + z^{-4} + z^{-7}$.

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SOLUTION

(a) Given: $y(n) = x(n) + 2x(n-2) + x(n-4)$ (diff. eq.)

Impulse response: $h(n) = \delta(n) + 2\delta(n-2) + \delta(n-4)$

Frequency response: $H(e^{j\hat{\omega}}) = 1 + 2e^{-j2\hat{\omega}} + e^{-j4\hat{\omega}}$

System function: $H(z) = 1 + 2z^{-2} + z^{-4}$

(b) Given: $h(n) = \delta(n) + \delta(n-1) + \delta(n-2) + \delta(n-3) + \delta(n-4)$ (Imp. resp.)

Difference Eq: $y(n) = x(n) + x(n-1) + x(n-2) + x(n-3) + x(n-4)$

Frequency resp: $H(e^{j\hat{\omega}}) = 1 + e^{-j\hat{\omega}} + e^{-j2\hat{\omega}} + e^{-j3\hat{\omega}} + e^{-j4\hat{\omega}}$

System function: $H(z) = 1 + z^{-1} + z^{-2} + z^{-3} + z^{-4}$

(c) Given: $H(e^{j\hat{\omega}}) = [1 + \cos(2\hat{\omega})]e^{-j\hat{\omega}3}$ (freq. resp.)

$$H(e^{j\hat{\omega}}) = \left(1 + \frac{e^{j2\hat{\omega}} + e^{-j2\hat{\omega}}}{2}\right)e^{-j3\hat{\omega}} =$$

$$= e^{-j3\hat{\omega}} + \frac{1}{2}e^{-j\hat{\omega}} + \frac{1}{2}e^{-j5\hat{\omega}} =$$

$$= \frac{1}{2}e^{-j\hat{\omega}} + e^{-j3\hat{\omega}} + \frac{1}{2}e^{-j5\hat{\omega}} \Rightarrow$$

System function: $H(z) = \frac{1}{2}z^{-1} + z^{-3} + \frac{1}{2}z^{-5}$

Impulse Response: $h(n) = \frac{1}{2}\delta(n-1) + \delta(n-3) + \frac{1}{2}\delta(n-5)$

Difference Eq: $y(n) = \frac{1}{2}x(n-1) + x(n-3) + \frac{1}{2}x(n-5)$

(d) Given: $H(z) = 1 - 2z^{-2} + z^{-4} + z^{-7}$

Frequency Response: $H(e^{j\hat{\omega}}) = H(z)\big|_{z=e^{j\hat{\omega}}} \Rightarrow$

$$H(e^{j\hat{\omega}}) = 1 - 2e^{-j2\hat{\omega}} + e^{-j4\hat{\omega}} + e^{-j7\hat{\omega}}$$

Impulse response: $h(n) = \delta(n) - 2\delta(n-2) + \delta(n-4) + \delta(n-7)$

Difference Eq: $y(n) = x(n) - 2x(n-2) + x(n-4) + x(n-7)$

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