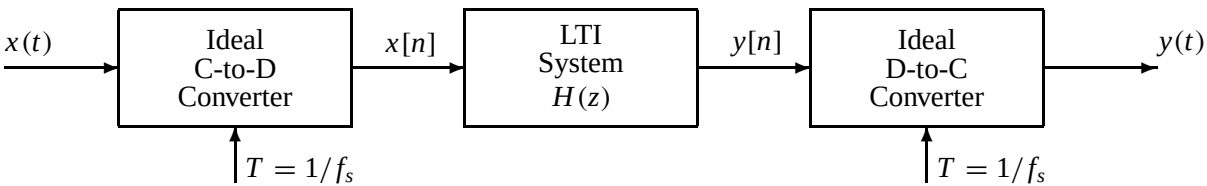
PROBLEM:

The input to the C-to-D converter in the figure below is

$$x(t) = 10 + 6\cos(2000\pi t + \pi/8) + 4\cos(6000\pi t - \pi/3)$$

The system function of the LTI system is

$$H(z) = (1 - z^{-2})$$

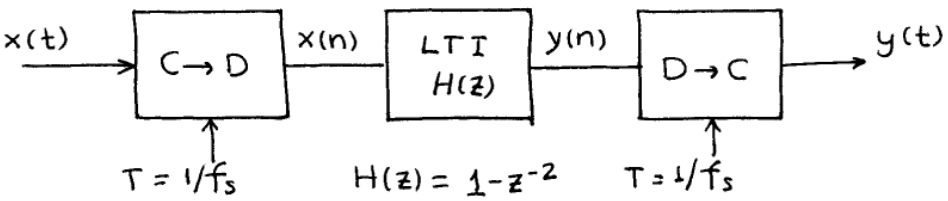


(a) If $f_s = 10000$ samples/second, determine an expression for $y(t)$, the output of the D-to-C converter.

(b) If $f_s = 5000$ samples/second, determine an expression for $y(t)$, the output of the D-to-C converter. *Note that even when aliasing distortion occurs, we can still determine the effect of the system on the input $x[n]$ and therefore we can determine $y(t)$ from $y[n]$.*

McClellan, Schafer and Yoder, *Signal Processing First*, ISBN 0-13-065562-7.
Prentice Hall, Upper Saddle River, NJ 07458.

SOLUTION



$$x(t) = 10 + 6\cos(2000\pi t + \pi/8) + 4\cos(6000\pi t - \pi/3)$$

(a) $f_s = 10000\,\text{Hz} > 2f_{max} = 2 \cdot 3000 = 6000\,\text{Hz}$ (no aliasing expected)

$$x(n) = x(t)\Big|_{t=nT} = 10 + 6\cos\left(\frac{2000}{10000}\pi n + \frac{\pi}{8}\right) + 4\cos\left(\frac{6000}{10000}\pi n - \pi/3\right)$$

$$= 10 + 6\cos(0.2\pi n + \pi/8) + 4\cos(0.6\pi n - \pi/3)$$

Frequency response $H(\hat{\omega}) = 1 - e^{-j2\hat{\omega}}$

$$H(\hat{\omega} = 0) = 1 - e^{-j2\cdot 0} = 1 - 1 = 0$$

$$H(\hat{\omega} = 0.2\pi) = 1 - e^{-j2\cdot 0.2\pi} = 1 - e^{-j0.4\pi} = 0.690983 + j\,0.951056$$

$$= 1.175571\, e^{+j54^\circ} = 1.175571\, e^{j0.3\pi}$$

$$H(\hat{\omega} = 0.6\pi) = 1 - e^{-j2\cdot 0.6\pi} = 1 - e^{-j1.2\pi} = 1 - e^{+j0.8\pi} =$$

$$= 1 - (-0.80901699 + j\,0.58778525) =$$

$$= 1.80901699 - j\,0.58778525 =$$

$$= 1.902113\, e^{-j18^\circ} = 1.902113\, e^{-j0.1\pi}$$

$$y(n) = 0 \cdot 10 + 1.175571 \cdot 6 \cdot \cos(0.2\pi n + \pi/8 + 0.3\pi) +$$

$$1.902113 \cdot 4 \cdot \cos(0.6\pi n - \pi/3 - 0.1\pi) =$$

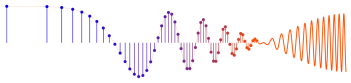
$$= 7.0534 \cdot \cos(0.2\pi n + 0.425\pi) + 7.6085\cos(0.6\pi n - 0.4333\pi)$$

$$y(t) = y(n)\Big|_{n=tf_s} = 7.0534\cos(0.2\pi\, 10000\, t + 0.425\pi) +$$

$$7.6085\cos(0.6\pi\, 10000\, t - 0.4333\pi) =$$

$$= 7.0534\cos(2000\pi t + 0.425\pi) +$$

$$7.6085\cos(6000\pi t - 0.4333\pi)$$



(b) $f_s = 5000\,\text{Hz} < 2f_{max} = 2 \cdot 3000 = 6000\,\text{Hz}$ (aliasing expected).

$$x(n) = 10 + 6\cos\left(\frac{2000\pi}{5000}n + \frac{\pi}{8}\right) + 4\cos\left(\frac{6000\pi}{5000}n - \frac{\pi}{3}\right) =$$

$$= 10 + 6\cos(0.4\pi n + \pi/8) + 4\cos(1.2\pi n - \pi/3)$$

$$= 10 + 6\cos(0.4\pi n + \pi/8) + 4\cos((1.2\pi - 2\pi)n - \pi/3) =$$

$$= 10 + 6\cos(0.4\pi n + \pi/8) + 4\cos(0.8\pi n + \pi/3)$$

$$H(\hat{\omega} = 0) = 0$$

$$H(\hat{\omega} = 0.4\pi) = 1 - e^{-j2\,0.4\pi} = 1.9021\,e^{-j18^\circ} = 1.9021\,e^{j0.1\pi}$$

$$H(\hat{\omega} = 0.8\pi) = 1 - e^{-j2\cdot 0.8\pi} = 1.1756\,e^{-j54^\circ} = 1.1756\,e^{-j0.3\pi}$$

$$y(n) = \emptyset \cdot 10 + 1.9021 \cdot 6\cos(0.4\pi n + \pi/8 + 0.1\pi) +$$

$$1.1756 \cdot 4 \cdot \cos(0.8\pi n + \pi/3 - 0.3\pi) =$$

$$= 11.4126\cos(0.4\pi n + 0.225\pi) + 4.7024\cos(0.8\pi n + 0.033\pi)$$

$$y(t) = 11.4126\cos(0.4\pi\,5000t + 0.225\pi) + 4.7024\cos(0.8\pi\,5000t + 0.033\pi)$$

$$= 11.4126\cos(2000\pi t + 0.225\pi) + 4.7024\cos(4000\pi t + 0.033\pi)$$