PROBLEM:

A linear time-invariant filter is described by the difference equation

$$y[n] = x[n] + x[n-1] + x[n-2] + x[n-3] + x[n-4] = \sum_{k=0}^{4} x[n-k]$$

(a) What is the impulse response, $h[n]$, of this system?

(b) Show that $H(z)$ for this system can be expressed as follows:

$$H(z) = \frac{1 - z^{-5}}{1 - z^{-1}}$$

(c) Plot the zeros of $H(z)$ in the complex z-plane. *Hint: Remember the N-th roots of unity.*

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SOLUTION

(a) $$y(n) = \sum_{k=0}^{4} x(n-k)$$

$$h(n) = \sum_{k=0}^{4} \delta(n-k) = \delta(n) + \delta(n-1) + \delta(n-2) + \delta(n-3) + \delta(n-4)$$

(b) $$H(z) = 1 + z^{-1} + z^{-2} + z^{-3} + z^{-4}$$

Remember the useful identity: $1 + x + x^2 + \cdots + x^M = \dfrac{1 - x^{M+1}}{1-x}$

For $x = z^{-1}$ we get

$$H(z) = \frac{1 - z^{-5}}{1 - z^{-1}}$$

(c) $$H(z) = \frac{z^5 - 1}{z^4(z-1)}$$

$H(z) = 0 \Rightarrow \quad z^5 = 1$ and $z \neq 1$ since $z = 1$ is also a pole

and cancels with the zero at $z = 1$.

Solutions of $z^5 = 1$:

$$z^5 = 1 = e^{j2\pi m} \qquad m = \text{integer}$$

$$z = e^{j\frac{2\pi}{5}m} \qquad m = 0, 1, 2, 3, 4 \qquad (\text{for } m \geq 5 \text{ the same roots repeat})$$

Since $z = 1$ must be excluded $m = 1, 2, 3, 4$

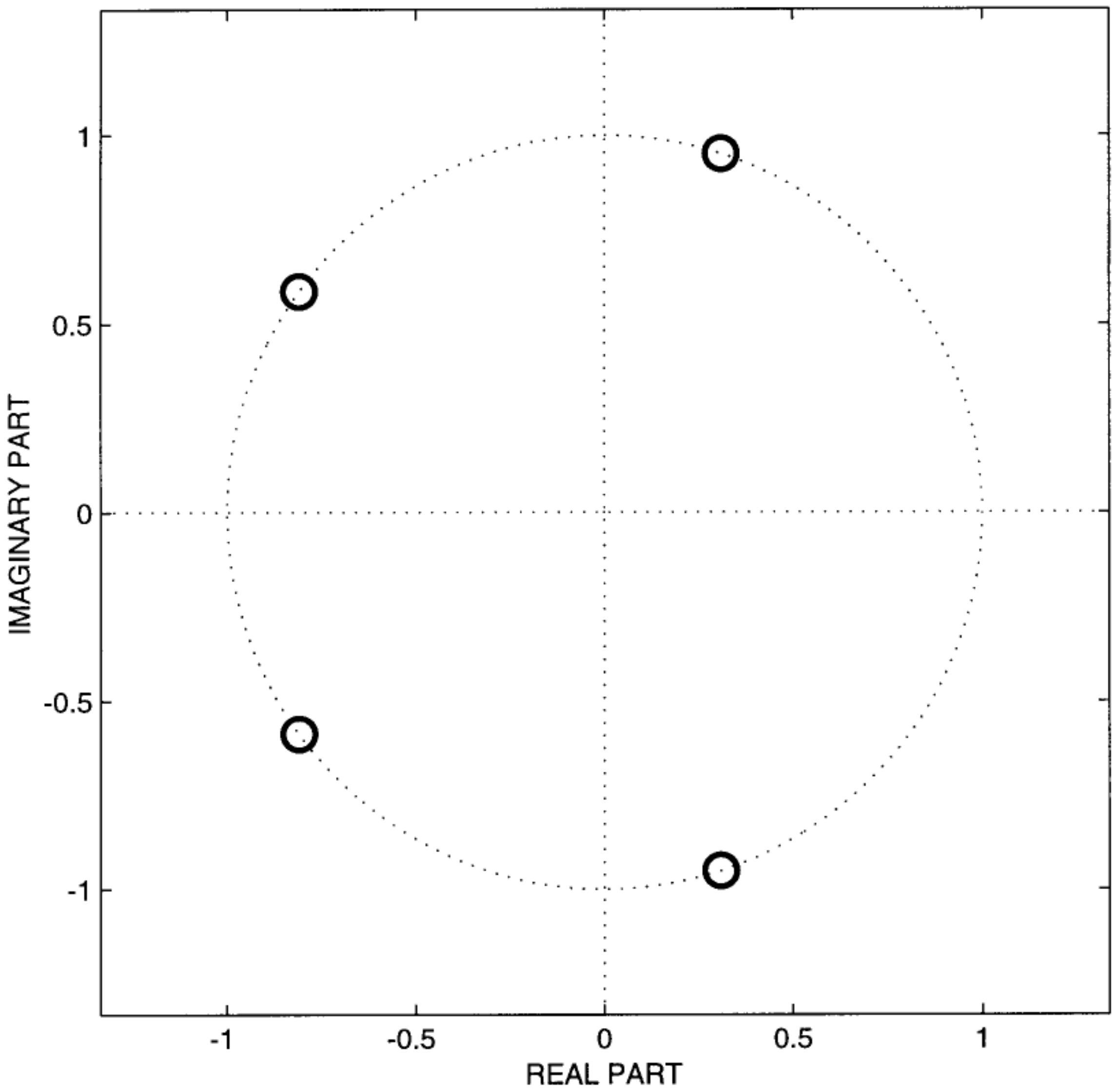
$$z_1 = e^{j2\pi/5}$$

$$z_2 = e^{j4\pi/5}$$

$$z_3 = e^{j6\pi/5}$$

$$z_4 = e^{j8\pi/5}$$

Using MATLAB's zplane (or DSPFIRST zzplane) the plot of zeros is shown in the next page.



Zeros of $H(z) = \dfrac{1 - z^{-5}}{1 - z^{-1}}$

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