PROBLEM:

Try your hand at expressing each of the following in a simpler form:

(a) $[2\delta(t-2) + 2e^{-t}u(t) + \cos(100\pi t)u(t)] * \delta(t-5) =$

(b) $[\delta(t-.002) + \delta(t+.002)]\cos(100\pi t)u(t) =$

(c) $\dfrac{d}{dt}\left\{e^{-2t}[u(t) - u(t-2)]\right\} =$

(d) $\displaystyle\int\limits_{-\infty}^{t} [\delta(\tau-.002) + \delta(\tau+.002)]\cos(100\pi\tau)u(\tau)d\tau =$

Note: use properties of the impulse signal $\delta(t)$ and the unit-step signal $u(t)$ to perform the simplifications. For example, recall

$$\delta(t) = \frac{d}{dt}u(t) \qquad \text{where} \qquad u(t) = \int_{-\infty}^{t} \delta(\tau)d\tau = \begin{cases} 1 & \text{for } t \geq 0 \\ 0 & \text{for } t < 0 \end{cases}$$

Be careful to distinguish between multiplication and convolution. Convolution is denoted by a “star”, as in $x(t) * \delta(t-2) = x(t-2)$ and multiplication is usually indicated as in $x(t)\delta(t-2) = x(2)\delta(t-2)$.

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SOLUTION

(a) $2\delta(t-2) * \delta(t-5) + 2e^{-t}u(t) * \delta(t-5) + \cos(100\pi t)u(t) * \delta(t-5)$

$$= 2\delta(t-7) + 2e^{-(t-5)}u(t-5) + \cos(100\pi(t-5))\,u(t-5)$$

Note: used the shifting property: $x(t)*\delta(t-5) = x(t-5)$

(b) $\delta(t-0.002)\cos(100\pi t)u(t) + \delta(t+0.002)\cos(100\pi t)u(t)$

Note: use the sampling property: $f(t)\delta(t-a) = f(a)\delta(t-a)$

$$\underbrace{\cos(100\pi(0.002))}_{=\cos(0.2\pi)}\,\underbrace{u(0.002)}_{=1}\,\delta(t-0.002) + \cos(100\pi(-0.002))\,\underbrace{u(-0.002)}_{=0}\,\delta(t+0.002)$$

$$= \cos(0.2\pi)\,\delta(t-0.002)$$

(c) Take derivative with the product rule:

$$\frac{d}{dt}e^{-2t}u(t) - \frac{d}{dt}e^{-2t}u(t-2)$$

$$= -2e^{-2t}u(t) + \underbrace{e^{-2t}\delta(t)}_{e^{-0}\delta(t)} - \left(-2e^{-2t}u(t-2) + \underbrace{e^{-2t}\delta(t-2)}_{e^{-4}\delta(t-2)}\right)$$

$$= -2e^{-2t}u(t) + \delta(t) + 2e^{-2t}u(t-2) - e^{-4}\delta(t-2)$$

(d) $\displaystyle\int_{-\infty}^{t} \underbrace{\delta(\tau-0.002)\cos(100\pi\tau)u(\tau)}_{\text{eval @ } \tau=0.002}\,d\tau + \int_{-\infty}^{t} \underbrace{\delta(\tau+0.002)\cos(100\pi\tau)u(\tau)}_{\text{eval at } \tau=-0.002}\,d\tau$

$u(-0.002) = 0$

$$\int_{-\infty}^{t} \cos(0.2\pi)\,\delta(\tau-0.002)\,d\tau + 0$$

$$= \cos(0.2\pi)\; u(\tau-0.002)\Big|_{-\infty}^{t}$$

$$= \cos(0.2\pi)\,u(t-0.002)$$

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