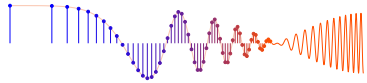


PROBLEM:

A linear time-invariant system has impulse response:

$$h(t) = e^{-0.2(t-1)}[u(t-1) - u(t-8)] = \begin{cases} e^{-0.2(t-1)} & 1 \le t < 8 \\ 0 & \text{otherwise} \end{cases}$$

(a) Plot $h(t-\tau)$ as a function of τ for $t = -1$, 2, and 15.

(b) Find the output $y(t)$ when the input is $x(t) = \delta(t+1)$.

(c) Use the convolution integral to determine the output $y(t)$ when the input is

$$x(t) = e^{-0.25t}[u(t) - u(t-10)] = \begin{cases} e^{-0.25t} & 0 \le t < 10 \\ 0 & \text{otherwise} \end{cases}$$

McClellan, Schafer and Yoder, *Signal Processing First*, ISBN 0-13-065562-7.
Prentice Hall, Upper Saddle River, NJ 07458.

SOLUTION

(a) see MATLAB plots

(b) $y(t) = x(t) * h(t) = \delta(t+1) * h(t) = h(t+1)$

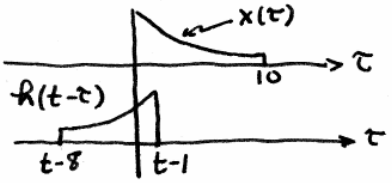
$$h(t+1) = e^{-0.2t}\left[u(t) - u(t-7)\right]$$

(c) $$y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$ ← FLIP & SLIDE $h(\cdot)$

use the plots from part (a) as examples to get the 5 regions

Region I (no overlap)

$y(t) = 0$ when $t-1 < 0$, or $t < 1$



Region II (partial overlap)

when $t-1 \geq 0$ & $t-8 < 0$ → $1 \leq t < 8$

$$y(t) = \int_0^{t-1} e^{-0.25\tau}\, e^{-0.2(t-\tau-1)}\, d\tau = \int_0^{t-1} e^{-0.2(t-1)}\, e^{-0.05\tau}\, d\tau$$

$$y(t) = e^{-0.2(t-1)} \left.\frac{e^{-0.05\tau}}{-0.05}\right|_0^{t-1}$$

$$= \frac{e^{-0.2(t-1)}}{-0.05}\left(e^{-0.05(t-1)} - 1\right) = -20e^{-0.25(t-1)} + 20e^{-0.2(t-1)}$$

Region III (complete overlap when $t-1 \leq 10$ & $t-8 \geq 0$)

when $8 \leq t \leq 11$

$$y(t) = \int_{t-8}^{t-1} (\text{same integrand})\, d\tau = e^{-0.2(t-1)} \left.\frac{e^{-0.05\tau}}{-0.05}\right|_{t-8}^{t-1}$$

$$= -20e^{-0.2(t-1)}\left(e^{-0.05(t-1)} - e^{-0.05(t-8)}\right)$$

Region IV (partial overlap when $t-1 > 10$ & $t-8 \leq 10$)

when $11 < t \leq 18$

$$y(t) = \int_{t-8}^{10} (\text{same integrand})\, d\tau = e^{-0.2(t-1)} \left.\frac{e^{-0.05\tau}}{-0.05}\right|_{t-8}^{10}$$

$$= -20e^{-0.2(t-1)}\left(e^{-0.5} - e^{-0.05(t-8)}\right)$$

Region V (no overlap when $t-8 > 10$)

when $t > 18$ $\quad y(t) = 0$