PROBLEM:

A linear time-invariant system has impulse response: $h(t) = e^{-(t-1)}u(t-1)$

(a) Plot $h(t-\tau)$ versus τ, for $t = -3$ and $t = 2$. Label your plot.

(b) Is the LTI system causal? Give a reason to support your answer.

(c) Is the system stable? Explain with a proof or counter-example.

(d) If the input is $x(t) = u(t+2)$, then it will be true that the output $y(t)$ is zero for $t \le t_1$. Find t_1.

(e) The rest of the output signal (for $t > t_1$) is non-zero, when the input is $x(t) = u(t+2)$. Use the convolution integral to find the non-zero portion of the output, i.e., find $y(t)$ for $t > t_1$.
Hint: it might be easier to flip and slide $x(t)$.

McClellan, Schafer and Yoder, *Signal Processing First*, ISBN 0-13-065562-7.
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SOLUTION

(a) see MATLAB plot

(b) An LTI system is causal if $h(t)=0$ for $t<0$

Yes, this system is causal because $u(t-1)=0$ for $t<1$

(c) An LTI system is stable if $\int_{-\infty}^{\infty}|h(t)|dt<\infty$

$$\int_{-\infty}^{\infty}|h(t)|dt=\int_{-\infty}^{\infty}\left|e^{-(t-1)}u(t-1)\right|dt$$

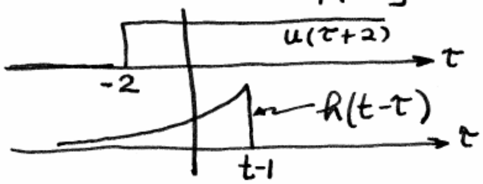
Yes this system is stable

$$=\int_{1}^{\infty}e^{-(t-1)}dt=\left.\frac{e^{-(t-1)}}{-1}\right|_{1}^{\infty}=-\left(0-e^{-0}\right)=+1<\infty$$

(d) $y(t)=x(t)*h(t)$

Thus we draw a "flip and slide" picture.

In this case flipping $h(t)$ shows the regions



The output $y(t)$ is zero when there is no overlap, i.e., when

$t-1\le-2 \Rightarrow$ $\boxed{t\le-1}$

(e) Partial overlap when $t-1>-2 \Rightarrow t>-1$

$$y(t)=\int_{-2}^{t-1}e^{-(t-\tau-1)}d\tau$$

$$=e^{-(t-1)}\int_{-2}^{t-1}e^{\tau}d\tau=e^{-(t-1)}\left.e^{\tau}\right|_{-2}^{t-1}=e^{-(t-1)}\left(e^{t-1}-e^{-2}\right)$$

$$y(t)=1-e^{-t-1}\quad\text{for } t>-1$$

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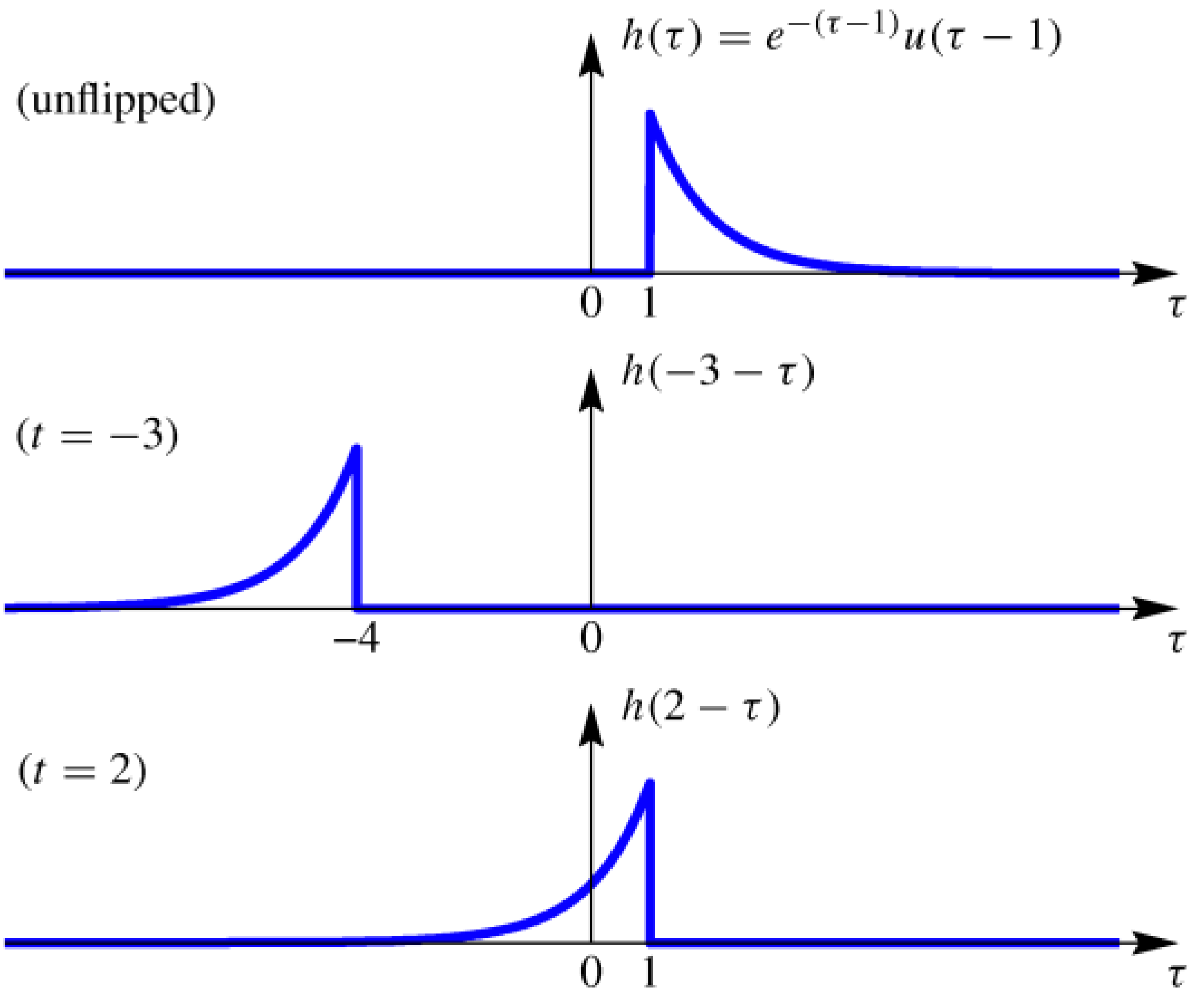
(a)



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