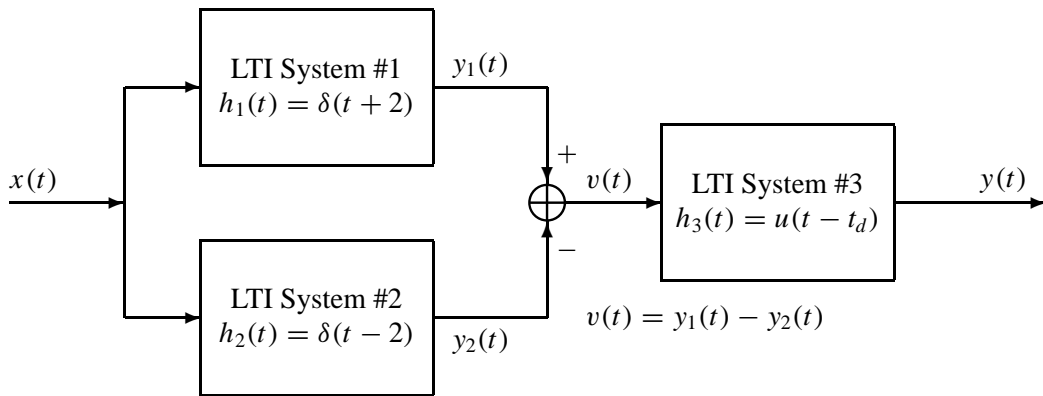


PROBLEM:



- What is the impulse response of the overall LTI system (i.e., from $x(t)$ to $y(t)$)? Give your answer both as an equation and as a carefully labeled sketch.
- How should the time delay t_d be chosen so that the overall system is causal?
- Which systems (#1, #2, #3) are stable? Is the overall system a stable system? Explain.



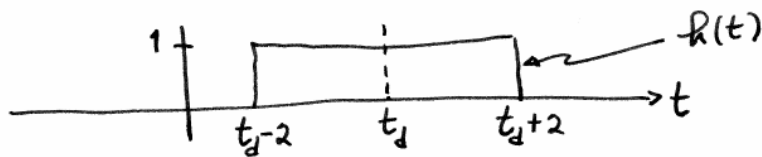
(a)

$$v(t) = y_1(t) - y_2(t) \quad \text{and} \quad y(t) = v(t) * u(t - t_d)$$

When $x(t) = \delta(t)$

$$v(t) = h_1(t) - h_2(t) \quad \text{and} \quad h(t) = (h_1(t) - h_2(t)) * u(t - t_d)$$

$$\begin{aligned} \text{Thus, } h(t) &= (\delta(t+2) - \delta(t-2)) * u(t - t_d) \\ &= u(t+2-t_d) - u(t-2-t_d) \end{aligned}$$



(b) For causality we need $h(t) = 0$ for $t < 0$.

$$\text{Thus } t_d - 2 > 0 \Rightarrow t_d > 2$$

(c) For stability we need $\int_{-\infty}^{\infty} |h(t)| dt < \infty$

$$\int_{-\infty}^{\infty} |h_1(t)| dt = \int_{-\infty}^{\infty} \delta(t+2) dt = 1 \quad \text{--- System \# 1 is stable}$$

$$\int_{-\infty}^{\infty} |h_2(t)| dt = \int_{-\infty}^{\infty} \delta(t-2) dt = 1 \quad \text{--- System \# 2 is stable}$$

$$\int_{-\infty}^{\infty} |h_3(t)| dt = \int_{-\infty}^{\infty} |u(t - t_d)| dt = \int_{t_d}^{\infty} dt = t \Big|_{t_d}^{\infty} \rightarrow \infty$$

\therefore System # 3 is unstable

For the overall system:

$$\int_{-\infty}^{\infty} |h(t)| dt = \int_{t_d-2}^{t_d+2} 1 dt = t \Big|_{t_d-2}^{t_d+2} = (t_d+2) - (t_d-2) = 4$$

\therefore The overall system is stable