PROBLEM:

The impulse response of a continuous-time system is

$$h(t) = \delta(t) + e^{-t}u(t).$$

Determine $y(t) = x(t) * h(t)$, the output of this system when the input is

$$x(t) = \delta(t) - e^{-2t}u(t).$$

Hint: Express $y(t)$ as the sum of four convolutions, three of which will be convolutions of $\delta(t)$ with another function. (Remember that $\delta(t) * v(t) = v(t)$, where $v(t)$ is any continuous-time signal.) The fourth convolution will be between two exponential signals.

McClellan, Schafer and Yoder, *Signal Processing First*, ISBN 0-13-065562-7.
Prentice Hall, Upper Saddle River, NJ 07458.

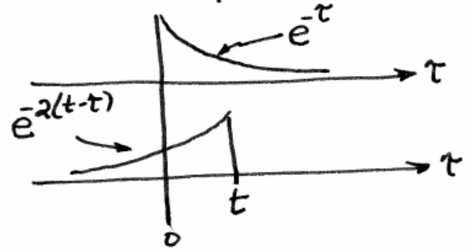
SOLUTION

$$
\begin{aligned}
y(t) &= x(t) * h(t) \\
&= \left(\delta(t) - e^{-2t}u(t)\right) * \left(\delta(t) + e^{-t}u(t)\right) \\
&= \delta(t)*\delta(t) - e^{-2t}u(t)*\delta(t) + \delta(t)*e^{-t}u(t) - e^{-2t}u(t)*e^{-t}u(t) \\
&= \delta(t) - e^{-2t}u(t) + e^{-t}u(t) - \underbrace{e^{-2t}u(t)*e^{-t}u(t)}
\end{aligned}
$$

This term is hard.
Use flip-and-slide pictures

There are 2 regions
No overlap when $t<0$

Partial overlap when $t \geq 0$



$$
y(t) = \int_0^t e^{-\tau} e^{-2(t-\tau)}\, d\tau
$$

$$
= e^{-2t}\int_0^t e^{\tau} d\tau = e^{-2t}\left(e^{t}-1\right) = e^{-t} - e^{-2t} \quad \text{for } t \geq 0
$$

A concise way to write this result is

$$
e^{-2t}u(t) * e^{-t}u(t) = e^{-t}u(t) - e^{-2t}u(t)
$$

$$
y(t) = \delta(t) - e^{-2t}u(t) + e^{-t}u(t) - \left(e^{-t}u(t) - e^{-2t}u(t)\right)
$$

$$
\boxed{y(t) = \delta(t)}
$$

All the exponential terms cancel out.