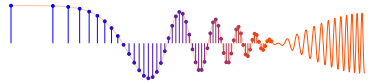

PROBLEM:

The delay property of Fourier transform states that if $X(j\omega)$ is the Fourier transform of $x(t)$, then the Fourier transform of $x(t-t_d)$ is $e^{-j\omega t_d}X(j\omega)$, i.e.,

$$x(t-t_d) \Longleftrightarrow e^{-j\omega t_d}X(j\omega).$$

Use this property to find the Fourier transforms of the following signals. Simplify your answer as much as possible.

(a) $x(t) = -\delta(t) + 2\delta(t-T) - \delta(t-2T)$

(b) $x(t) = 10\dfrac{\sin(400\pi(t-0.1))}{\pi(t-0.1)}$

(c) $x(t) = 10e^{-2t}u(t) - 10e^{-2t}u(t-3)$

McClellan, Schafer and Yoder, *Signal Processing First*, ISBN 0-13-065562-7.
Prentice Hall, Upper Saddle River, NJ 07458.

SOLUTION

$$x(t-t_d) \Leftrightarrow e^{-j\omega t_d} \; X(j\omega)$$

a) $x(t) = -\delta(t) + 2\delta(t-T) - \delta(t-2T)$

$$-\delta(t) \Leftrightarrow 1$$

$$2\delta(t-T) \Leftrightarrow 2e^{-j\omega T}$$

$$-\delta(t-2T) \Leftrightarrow -e^{-j\omega 2T}$$

$$X(j\omega) = -1 + 2e^{-j\omega T} - e^{-j\omega 2T}$$

b) $$x(t) = 10 \cdot \frac{\sin(400\pi(t-0.01))}{\pi(t-0.01)}$$

$$x_1(t) = \frac{\sin(400\pi t)}{\pi t} \Leftrightarrow X_1(j\omega) = u(\omega+400\pi) - u(\omega-400\pi)$$

$$x(t) = 10\,x_1(t-0.1)$$

$$\Leftrightarrow X(j\omega) = 10e^{-j\omega 0.1}\left[u(\omega+400\pi) - u(\omega-400\pi)\right]$$

c) $$x(t) = 10e^{-2t}u(t) - 10e^{-2t}u(t-3)$$

note: $$-10e^{-2t}u(t-3) = -10e^{-2(t-3)}e^{-6}u(t-3)$$

$$X(j\omega) = \frac{10}{2+j\omega} - 10e^{-6}\frac{e^{-j\omega 3}}{2+j\omega} = \frac{10(1-e^{-j\omega 3-6})}{2+j\omega}$$