PROBLEM:

Determine the z-transforms of the following. *Express your answer as the ratio of polynomials in z^{-1} by placing all terms over a common denominator.*

(a) $x_a[n] = \delta[n] - (-0.8)^n u[n]$.

(b) $x_b[n] = 2(\frac{1}{2})^n u[n] + 2(-\frac{1}{2})^n u[n]$.

(c) $x_b[n] = \delta[n-1] + u[n]$.

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SOLUTION

(a) $x_a(n) = \delta(n) - (-0.8)^n u(n)$

$$a^n u(n) \longleftrightarrow \frac{1}{1 - a z^{-1}} \quad \text{if} \quad |z| > |a| \qquad \delta(n) \longleftrightarrow 1$$

$$X_a(z) = 1 - \frac{1}{1-(-0.8)z^{-1}} = 1 - \frac{1}{1+0.8z^{-1}} = \frac{0.8z^{-1}}{1+0.8z^{-1}} \quad (|z| > 0.8)$$

(b) $x_b(n) = 2\left(\frac{1}{2}\right)^n u(n) + 2\left(-\frac{1}{2}\right)^n u(n)$

$$X_b(z) = 2\,\frac{1}{1-\frac{1}{2}z^{-1}} + 2\,\frac{1}{1-(-\frac{1}{2})z^{-1}} \qquad \left(|z| > \tfrac{1}{2}\right)$$

$$= \frac{2}{1-\frac{1}{2}z^{-1}} + \frac{2}{1+\frac{1}{2}z^{-1}} = 2\,\frac{1+\frac{1}{2}z^{-1} + 1 - \frac{1}{2}z^{-1}}{\left(1-\frac{1}{2}z^{-1}\right)\left(1+\frac{1}{2}z^{-1}\right)} =$$

$$= \frac{4}{1 - \frac{1}{4}z^{-2}}$$

(c) $x_b(n) = \delta(n-1) + u(n)$

$$\text{if} \quad x(n) \longleftrightarrow X(z) \Rightarrow x(n-1) \longleftrightarrow z^{-1}X(z)$$

$$\delta(n) \longleftrightarrow 1 \Rightarrow \delta(n-1) \longleftrightarrow z^{-1}\cdot 1 = z^{-1}$$

$$u(n)\ ? \qquad \mathcal{U}(z) = \sum_{n=0}^{\infty} u(n) z^{-n} = \sum_{n=0}^{\infty} z^{-n} = \frac{1}{1-z^{-1}} \quad \text{if} \quad |z| > 1$$

$$X_b(z) = z^{-1} + \frac{1}{1-z^{-1}} = \frac{z^{-1}(1-z^{-1}) + 1}{1-z^{-1}} = \frac{1 + z^{-1} - z^{-2}}{1-z^{-1}}$$

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