PROBLEM:

Use the most convenient form of these system functions to determine the corresponding impulse responses of the following:

(a) $$H_a(z) = \frac{1+z^{-1}}{1+0.5z^{-1}} = \frac{1}{1+0.5z^{-1}} + \frac{z^{-1}}{1+0.5z^{-1}} = 2 - \frac{1}{1+0.5z^{-1}}.$$

(b) $$H_b(z) = \frac{2-0.9z^{-1}}{1-0.9z^{-1}+0.81z^{-2}} = \frac{1}{1-0.9e^{j\pi/3}z^{-1}} + \frac{1}{1-0.9e^{-j\pi/3}z^{-1}}.$$

(c) $$H_c(z) = \frac{1+z^{-2}}{1+0.25z^{-2}} = 4 - \frac{1.5}{1-0.5e^{j\pi/2}z^{-1}} - \frac{1.5}{1-0.5e^{-j\pi/2}z^{-1}}.$$

(d) $$H_d(z) = 1 + z^{-1} + z^{-2} + z^{-3} = \frac{1-z^{-4}}{1-z^{-1}}$$

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SOLUTION

(a) $$H_a(z) = \frac{1+z^{-1}}{1+0.5z^{-1}} = \frac{1}{1+0.5z^{-1}} + \frac{z^{-1}}{1+0.5z^{-1}} = 2 - \frac{1}{1+0.5z^{-1}}$$

$$h_a(n) = 2\,\delta(n) - (-0.5)^n u(n)$$

since $\delta(n) \longleftrightarrow 1$ $\qquad a^n u(n) \longleftrightarrow \dfrac{1}{1-\alpha z^{-1}}$

$h_a(n)$ can also be written as $\mathcal{Z}^{-1}\left\{ \dfrac{1}{1+0.5z^{-1}} + \dfrac{z^{-1}}{1+0.5z^{-1}} \right\}$

where $\mathcal{Z}^{-1}$ denotes inverse z-transform. In this case

$$\begin{aligned}
h_a(n) &= (-0.5)^n u(n) + (-0.5)^{n-1} u(n-1) = \\
&= 1\cdot\delta(n) + (-0.5)^n u(n-1) + (-0.5)^{n-1} u(n-1) = \\
&= \delta(n) + \left[(-0.5)^n + (-0.5)^{n-1}\right] u(n-1) = \\
&= \delta(n) - 0.5\left[(-0.5)^{n-1}\right] u(n-1) + (-0.5)^{n-1} u(n-1) = \\
&= \delta(n) + (1-0.5)\,(-0.5)^{n-1} u(n-1) = \\
&= \delta(n) - (-0.5)\,(-0.5)^{n-1} u(n-1) = \\
&= \delta(n) - (-0.5)^n u(n-1) = \\
&= \delta(n) + \delta(n) - \delta(n) - (-0.5)^n u(n-1) = \\
&= 2\,\delta(n) - (-0.5)^n u(n)
\end{aligned}$$

(b) $$H_b(z) = \frac{2-0.9z^{-1}}{1-0.9z^{-1}+0.81z^{-2}} = \frac{1}{1-0.9e^{j\pi/3}z^{-1}} + \frac{1}{1-0.9e^{-j\pi/3}z^{-1}}$$

$$\frac{1}{1-0.9e^{j\pi/3}z^{-1}} \longleftrightarrow (0.9e^{j\pi/3})^n u(n)$$

$$\frac{1}{1-0.9e^{-j\pi/3}z^{-1}} \longleftrightarrow (0.9e^{-j\pi/3})^n u(n)$$

$$\begin{aligned}
h_b(n) &= (0.9)^n \left[e^{j(\pi/3)n} + e^{-j(\pi/3)n} \right] u(n) = \\
&= 2\,(0.9)^n \cos\left(\frac{\pi}{3}n\right) u(n)
\end{aligned}$$

(c) $$H_c(z) = \frac{1+z^{-2}}{1+0.25z^{-2}} = 4 - \frac{1.5}{1-0.5e^{j\pi/2}z^{-1}} - \frac{1.5}{1-0.5e^{-j\pi/2}z^{-1}}$$

$$4 \longleftrightarrow 4\delta(n)$$

$$\frac{1.5}{1-0.5e^{j\pi/2}z^{-1}} \longleftrightarrow 1.5\,(0.5e^{j\pi/2})^n\,u(n)$$

$$\frac{1.5}{1-0.5e^{-j\pi/2}z^{-1}} \longleftrightarrow 1.5\,(0.5e^{-j\pi/2})^n\,u(n)$$

$$h_c(n) = 4\delta(n) - 1.5\,(0.5)^n\left[e^{j(\pi/2)n} + e^{-j(\pi/2)n}\right]u(n) =$$

$$= 4\delta(n) - 3\,(0.5)^n\cos\left(\frac{\pi}{2}\cdot n\right)u(n)$$

(d) $$H_d(z) = 1 + z^{-1} + z^{-2} + z^{-3} = \frac{1-z^{-4}}{1-z^{-1}}$$

$$h_d(n) = \delta(n) + \delta(n-1) + \delta(n-2) + \delta(n-3)$$

or by following the $\mathcal{Z}^{-1}\left\{\frac{1-z^{-4}}{1-z^{-1}}\right\} = \mathcal{Z}^{-1}\left\{\frac{1}{1-z^{-1}}\right\} - \mathcal{Z}^{-1}\left\{\frac{z^{-4}}{1-z^{-1}}\right\}$

$$\mathcal{Z}^{-1}\left\{\frac{1}{1-z^{-1}}\right\} = u(n)$$

$$\mathcal{Z}^{-1}\left\{z^{-4}\frac{1}{1-z^{-1}}\right\} = u(n-4)$$

Therefore $h_d(n) = u(n) - u(n-4) = \delta(n) + \delta(n-1) + \delta(n-2) + \delta(n-3)$

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