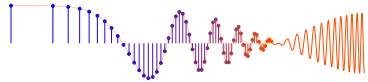


## PROBLEM:

For each of the difference equations below, determine the poles and zeros of the corresponding system function, $H(z)$. Plot the poles **(X)** and zeros **(O)** in the complex $z$-plane. In most cases, you should be able to obtain the poles and zeros by inspection, but you could use MATLAB's `roots` function to find them.

$$\mathcal{S}_1: \quad y[n] = -0.5y[n-1] + x[n] + x[n-1]$$

$$\mathcal{S}_2: \quad y[n] = 0.9y[n-1] - 0.81y[n-2] + 2x[n] - 0.9x[n-1]$$

$$\mathcal{S}_3: \quad y[n] = -0.25y[n-2] + x[n] + x[n-2]$$

$$\mathcal{S}_4: \quad y[n] = y[n-1] + x[n] - x[n-4]$$

McClellan, Schafer and Yoder, *Signal Processing First*, ISBN 0-13-065562-7.
Prentice Hall, Upper Saddle River, NJ 07458. 

SOLUTION

$S_1$ : $y(n) = -0.5\,y(n-1) + x(n) + x(n-1) \Rightarrow$

$Y(z) = -0.5\,z^{-1}\,Y(z) + X(z) + z^{-1}\,X(z) \Rightarrow$

$$\frac{Y(z)}{X(z)} = \frac{1 + z^{-1}}{1 + 0.5\,z^{-1}} \Rightarrow$$

$$H_1(z) = \frac{z + 1}{z + 0.5}$$

Zeros: $z + 1 = 0 \Rightarrow z = -1$

Poles : $z + 0.5 = 0 \Rightarrow z = -0.5$

$S_2$ : $y(n) = 0.9\,y(n-1) - 0.81\,y(n-2) + 2\,x(n) - 0.9\,x(n-1)$

$Y(z) = 0.9\,z^{-1}\,Y(z) - 0.81\,Y(z) + 2\,X(z) - 0.9\,z^{-1}\,X(z) \Rightarrow$

$$\frac{Y(z)}{X(z)} = \frac{2 - 0.9\,z^{-1}}{1 - 0.9\,z^{-1} + 0.81\,z^{-2}} \Rightarrow$$

$$H_2(z) = \frac{z\,[\,2z - 0.9\,]}{z^2 - 0.9z + 0.81}$$

Zeros: $z\,[2z - 0.9] = 0 \Rightarrow z = 0$

$z = 0.45$

Poles : $z^2 - 0.9z + 0.81 = 0 \Rightarrow z = 0.9\left\{\frac{1 + \sqrt{3}\,j}{2}\right\} = 0.9\,e^{j\pi/3}$

$z = 0.9\left\{\frac{1 - j\sqrt{3}}{2}\right\} = 0.9\,e^{-j\pi/3}$

$S_3$ : $y(n) = -0.25y(n-2) + x(n) + x(n-2) \Rightarrow$

$$Y(z) = -0.25z^{-2}Y(z) + X(z) + z^{-2}X(z) \Rightarrow$$

$$\frac{Y(z)}{X(z)} = \frac{1 + z^{-2}}{1 + 0.25z^{-2}} \Rightarrow$$

$$H_3(z) = \frac{z^2 + 1}{z^2 + 0.25}$$

Zeros: $z^2 + 1 = 0 \Rightarrow z^2 = -1 = e^{j\pi} = e^{j(\pi + 2\pi m)} \Rightarrow$

$$z_m = e^{j\frac{\pi + 2\pi m}{2}} \qquad m = 0, 1$$

$$z_0 = e^{j\pi/2} = j$$

$$z_1 = e^{j3\pi/2} = -j$$

Poles : $z^2 + 0.25 = 0 \Rightarrow z^2 = -0.25 = 0.25e^{j(\pi + 2\pi m)}$

$$z_m = 0.5e^{j\left(\frac{\pi + 2\pi m}{2}\right)} \qquad m = 0, 1$$

$$z_0 = 0.5e^{j\pi/2} = 0.5j$$

$$z_1 = 0.5e^{+j3\pi/2} = -0.5j$$

$S_4$ : $y(n) = y(n-1) + x(n) - x(n-4) \Rightarrow$

$$Y(z) = z^{-1}Y(z) + X(z) - z^{-4}X(z) \Rightarrow$$

$$\frac{Y(z)}{X(z)} = \frac{1 - z^{-4}}{1 - z^{-1}} \Rightarrow$$

$$H_4(z) = \frac{z^4 - 1}{z^3(z-1)}$$

Zeros: $z^4 - 1 = 0 \Rightarrow z^4 = 1 = e^{j2\pi m} \Rightarrow z = e^{j\frac{2\pi m}{4}} \qquad m = 0, 1, 2, 3$

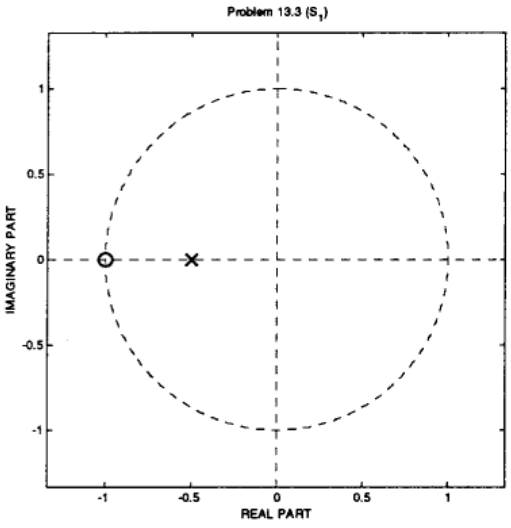
$z_0 = 1$, $z_1 = e^{j\pi/2} = j$, $z_2 = e^{j\pi} = -1$, $z_3 = e^{j\frac{3\pi}{2}} = -j$

Poles : $z^3(z-1) = 0 \Rightarrow z^3 = 0 \Rightarrow z = 0$ (triple)

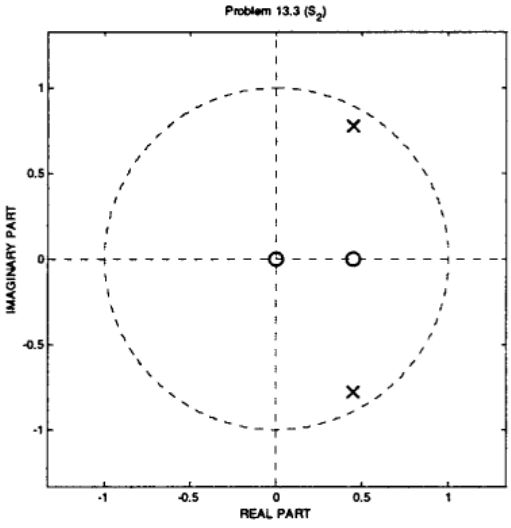
$z - 1 = 0 \Rightarrow z = 1$

All plots of zeros and poles are shown on page 6.

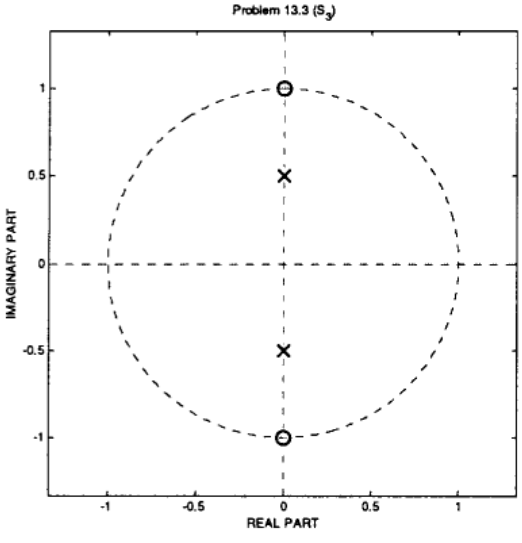
$S_1$




$S_2$




$S_3$




$S_4$