PROBLEM:

A causal LTI system has the following system function:

$$H(z) = \frac{1 + z^{-2}}{1 - 0.8z^{-1}}.$$

The following questions cover most of the ways available for analyzing IIR discrete-time systems.

(a) Plot the poles and zeros of $H(z)$ in the z-plane.

(b) Determine the difference equation that is satisfied by the general input $x[n]$ and the corresponding output $y[n]$ of the system.

(c) Use inverse z-transforms to determine the impulse response $h[n]$ of the system; i.e., the output of the system when the input is $x[n] = \delta[n]$.

(d) Determine if the system is stable.

(e) Determine an expression for the frequency response $H(e^{j\hat{\omega}})$ of the system.

(f) Use the frequency response function to determine the output $y_1[n]$ of the system when the input is

$$x_1[n] = 2\cos(0.5\pi n) \qquad -\infty < n < \infty.$$

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SOLUTION

$$H(z) = \frac{1 + z^{-2}}{1 - 0.8z^{-1}}$$

(a) $$H(z) = \frac{z^2 + 1}{z[z - 0.8]}$$

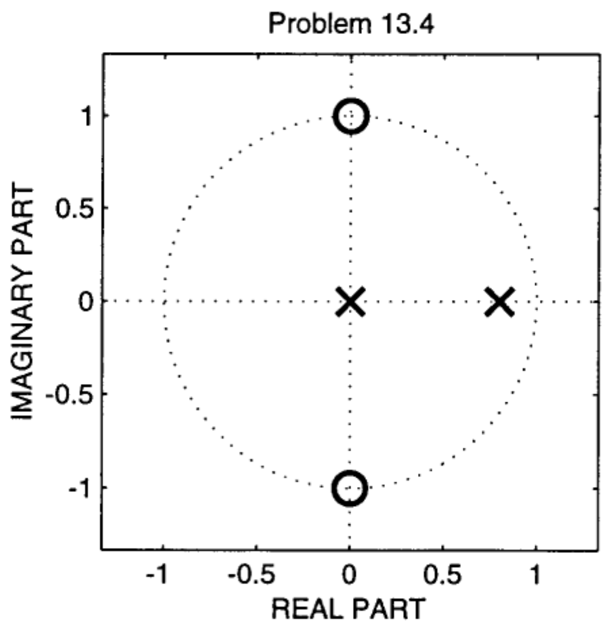
Zeros: $z^2 + 1 = 0 \Rightarrow z^2 = -1 = e^{j\pi} = e^{j(\pi + 2\pi m)} \Rightarrow$

$$z_m = e^{j\frac{\pi + 2\pi m}{2}} \qquad m = 0, 1$$

$$z_0 = e^{j\pi/2} = j$$

$$z_1 = e^{j3\pi/2} = -j$$

Poles: $z[z - 0.8] = 0 \Rightarrow z = 0$ & $z = 0.8$



(b) $H(z) = \frac{Y(z)}{X(z)} \Rightarrow$

$$\frac{Y(z)}{X(z)} = \frac{1 + z^{-2}}{1 - 0.8z^{-1}} \Rightarrow Y(z) - 0.8z^{-1}Y(z) = X(z) + z^{-2}X(z) \Rightarrow$$

$$y(n) - 0.8y(n-1) = x(n) + x(n-2) \Rightarrow$$

$$y(n) = 0.8y(n-1) + x(n) + x(n-2)$$

(c) $$H(z) = \frac{1+z^{-2}}{1-0.8z^{-1}} = \frac{1}{1-0.8z^{-1}} + z^{-2}\frac{1}{1-0.8z^{-1}}$$

$$\mathcal{Z}^{-1}\left\{\frac{1}{1-0.8z^{-1}}\right\} = 0.8^n u(n)$$

$$\mathcal{Z}^{-1}\left\{z^{-2}\frac{1}{1-0.8z^{-1}}\right\} = 0.8^{n-2} u(n-2)$$

$$h(n) = 0.8^n u(n) + 0.8^{n-2} u(n-2)$$

(d) Poles are 0, and 0.8 inside the unit circle

$\sim$ system is stable.

(e) $$H(e^{j\hat{\omega}}) = H(z)\Big|_{z=e^{j\hat{\omega}}} = \frac{1+e^{-j2\hat{\omega}}}{1-0.8e^{-j\hat{\omega}}}$$

(f) $$y_1(n) = H(e^{j(\hat{\omega}=\pi/2)})\, x_1(n)$$

$$H(e^{j(\pi/2)}) = \frac{1+e^{-j2\pi/2}}{1-0.8e^{-j\pi/2}} = 0$$

($\hat{\omega} = \pi/2$ zero of H)

Therefore $y_1(n) = 0$ for all n.

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