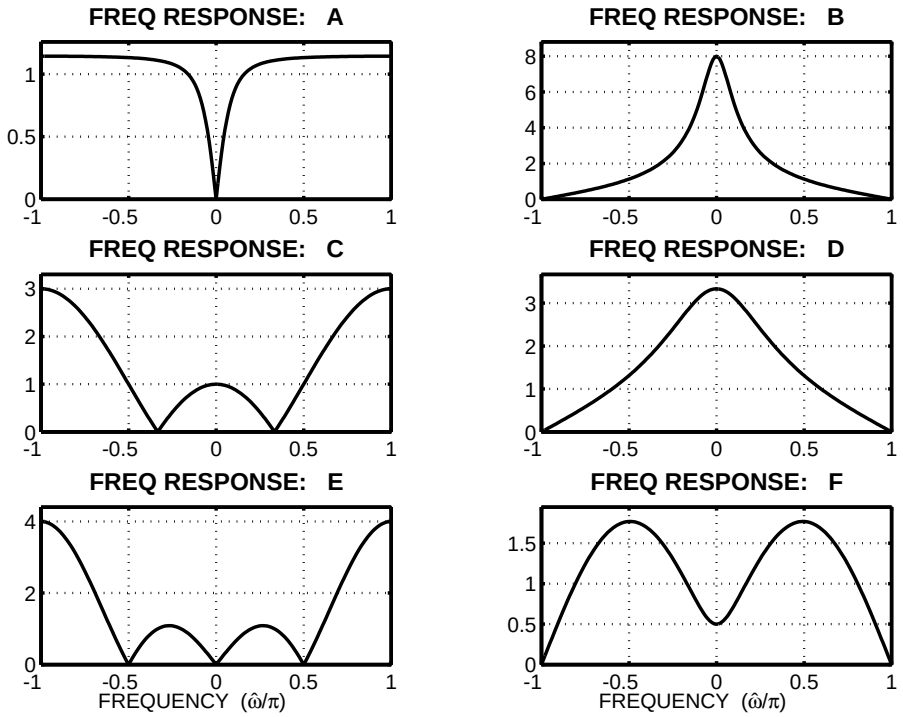
PROBLEM:



For each of the frequency response plots (A, B, C, D, E, F), determine which one of the following systems (specified by either an $H(z)$ or a difference equation) matches the frequency response (magnitude only). NOTE: frequency axis is **normalized**; it is $\hat{\omega}/\pi$. In addition, derive a formula for the magnitude-squared of the frequency response, $|H(e^{j\hat{\omega}})|^2$, for $\mathcal{S}_3$ and $\mathcal{S}_4$.

$\mathcal{S}_1$: $y[n] = 0.4y[n-1] + x[n] + x[n-1]$

$\mathcal{S}_2$: $H(z) = \dfrac{1+z^{-1}}{1-0.75z^{-1}}$

$\mathcal{S}_3$: $y[n] = -0.75y[n-1] + x[n] - x[n-1]$

$\mathcal{S}_4$: $H(z) = \dfrac{1-z^{-1}}{1-0.75z^{-1}}$

$\mathcal{S}_5$: $y[n] = x[n] - x[n-1] + x[n-2]$

$\mathcal{S}_6$: $H(z) = 1 - z^{-1} + z^{-2} - z^{-3}$

$\mathcal{S}_7$: $y[n] = x[n] + \frac{1}{4}x[n-1] - \frac{3}{4}x[n-2]$

$\mathcal{S}_8$: $H(z) = \frac{1}{3}(1-z^{-1})^3$

McClellan, Schafer and Yoder, *Signal Processing First*, ISBN 0-13-065562-7.
Prentice Hall, Upper Saddle River, NJ 07458.

SOLUTION

S_1 : $H(z) = \dfrac{1+z^{-1}}{1-0.4z^{-1}} \Rightarrow H(e^{j\hat{\omega}}) = \dfrac{1+e^{-j\hat{\omega}}}{1-0.4e^{-j\hat{\omega}}}$

S_2 : $H(z) = \dfrac{1+z^{-1}}{1-0.75z^{-1}} \Rightarrow H(e^{j\hat{\omega}}) = \dfrac{1+e^{-j\hat{\omega}}}{1-0.75e^{-j\hat{\omega}}}$

S_3 : $H(z) = \dfrac{1-z^{-1}}{1+0.75z^{-1}} \Rightarrow H(e^{j\hat{\omega}}) = \dfrac{1-e^{-j\hat{\omega}}}{1+0.75e^{-j\hat{\omega}}}$

S_4 : $H(z) = \dfrac{1-z^{-1}}{1-0.75z^{-1}} \Rightarrow H(e^{j\hat{\omega}}) = \dfrac{1-e^{-j\hat{\omega}}}{1-0.75e^{-j\hat{\omega}}}$

S_5 : $H(z) = 1 - z^{-1} + z^{-2} \Rightarrow H(e^{j\hat{\omega}}) = 1 - e^{-j\hat{\omega}} + e^{-j2\hat{\omega}}$

S_6 : $H(z) = 1 - z^{-1} + z^{-2} - z^{-3} \Rightarrow H(e^{j\hat{\omega}}) = 1 - e^{-j\hat{\omega}} + e^{-j2\hat{\omega}} - e^{-j3\hat{\omega}}$

S_7 : $H(z) = 1 + \frac{1}{4}z^{-1} - \frac{3}{4}z^{-2} \Rightarrow H(e^{j\hat{\omega}}) = 1 + \frac{1}{4}e^{-j\hat{\omega}} - \frac{3}{4}e^{-j2\hat{\omega}}$

S_8 : $H(z) = \frac{1}{3}(1-z^{-1})^3 \Rightarrow H(e^{j\hat{\omega}}) = \frac{1}{3}(1-e^{-j\hat{\omega}})^3$

(a) A : $H(\hat{\omega}=0) = H(z=1) = 0$

Only S_6, S_3, S_4, S_8 have $H(z=1) = 0$

$\hat{\omega} = \pi \rightsquigarrow \quad H_{S_6}(\hat{\omega}=\pi) = H_{S_6}(z=-1) = 4$

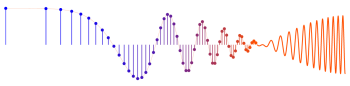
$\hat{\omega} = \pi \rightsquigarrow \quad H_{S_8}(\hat{\omega}=\pi) = H_{S_8}(z=-1) = \frac{8}{3} = 2.667$

$\hat{\omega}=\pi \rightsquigarrow \quad H_{S3}(\hat{\omega}=\pi) = H_{S3}(z=-1) = \dfrac{2}{1-0.75} = 8$

$\hat{\omega} = \pi \rightsquigarrow \quad H_{S4}(\hat{\omega}=\pi) = H_{S3}(z=-1) = \dfrac{2}{1.75} = 1.143$

Therefore

$$A \longleftrightarrow S_4$$



(b) B : $H(z=1) = 8 = H(\hat{\omega}=0)$

S_1 : $H(z=1) = \frac{2}{0.6} = 3.333$ $\quad H(z=-1) = 0$

S_2 : $H(z=1) = \frac{2}{0.25} = 8$ $\quad H(z=-1) = 0$

S_3 : $H(z=1) = \frac{1-1}{1.75} = 0$ $\quad H(z=-1) = \frac{2}{0.25} = 8$

S_4 : $H(z=1) = \frac{1-1}{1-0.75} = 0$ $\quad H(z=-1) = \frac{2}{1.75} = 1.143$

S_5 : $H(z=1) = 1$ $\quad H(z=-1) = 3$

S_6 : $H(z=1) = 0$ $\quad H(z=-1) = 4$

S_7 : $H(z=1) = \frac{2}{4} = 0.5$ $\quad H(z=-1) = 0$

S_8 : $H(z=1) = 0$ $\quad H(z=-1) = \frac{8}{3}$

Only S_2 matches response for $\hat{\omega}=0$ and $\hat{\omega}=\pm\pi$

$B \longleftrightarrow S_2$

(c) C : only S_5 matches response for $\hat{\omega}=0$, $\hat{\omega}=\pm\pi$

$C \longleftrightarrow S_5$

(d) D

only S_1 matches response for $\hat{\omega}=0$, $\hat{\omega}=\pm\pi$

(e) E : Only S_6 matches response for $\hat{\omega}=0$, $\hat{\omega}=\pm\pi$

(f) F : Only S_7 matches response for $\hat{\omega}=0$, $\hat{\omega}=\pm\pi$

Summarizing :

$A \longleftrightarrow S_4$ $\quad E \longleftrightarrow S_6$

$B \longleftrightarrow S_2$ $\quad F \longleftrightarrow S_7$

$C \longleftrightarrow S_5$

$D \longleftrightarrow S_1$