PROBLEM:

For both parts below draw a $\boxed{\text{phasor diagram}}$.

(a) Solve for $x[n]$ in the following equation:

$$x[n] - 3\cos(13n + 5\pi/3) = 5\cos(13n + 8\pi/3)$$

Express $x[n]$ in the form $x[n] = A\cos(\omega_0 n + \phi)$

(b) Use the idea of a "rotating phasor" to find a solution to

$$y[n] = \tfrac{1}{2}y[n-1] + 7\cos(\tfrac{1}{2}\pi n + \pi/4) \qquad \text{for all } n$$

Express the answer for $y[n]$ in the form $y[n] = A\cos(\omega_0 n + \phi)$. Do NOT assume that $y[n] = 0$ for $n < 0$.

Show the vector diagram of the phasor addition for the fixed value of $n = 0$.

McClellan, Schafer and Yoder, *Signal Processing First*, ISBN 0-13-065562-7.

SOLUTION

For both parts below draw a $\boxed{\text{phasor diagram}}$.

(a) Solve for $x[n]$ in the following equation:

$$x[n] - 3\cos(13n + 5\pi/3) = 5\cos(13n + 8\pi/3)$$

Express $x[n]$ in the form $x[n] = A\cos(\hat{\omega}_0 n + \phi)$

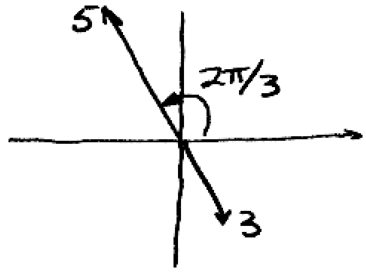
$x[n] = 3\cos(13n + 5\pi/3) + 5\cos(13n + 8\pi/3)$

CONVERT TO PHASORS:

$3\angle 5\pi/3 + 5\angle 8\pi/3$

$= 3\angle -\pi/3 + 5\angle 2\pi/3$

$= 2\angle 2\pi/3$



$\therefore$

$x[n] = 2\cos(13n + 2\pi/3)$

(b) Use the idea of a "rotating phasor" to find a solution to

$$y[n] = \tfrac{1}{2}y[n-1] + 7\cos(\tfrac{1}{2}\pi n + \pi/4) \qquad \text{for all } n$$

Express the answer for $y[n]$ in the form $y[n] = A\cos(\hat{\omega}_0 n + \phi)$. Do NOT assume that $y[n] = 0$ for $n < 0$.
Show the vector diagram of the phasor addition for the fixed value of $n = 0$.

ONLY POSSIBLE VALUE FOR $\hat{\omega}_0$ IS $\omega_0 = \pi/2$

$$Ae^{j\varphi}e^{j\hat{\omega}_0 n} = \tfrac{1}{2}Ae^{j\varphi}e^{j\hat{\omega}_0(n-1)} + 7e^{j(\omega_0 n + \pi/4)}.$$

$$\Rightarrow Ae^{j\varphi}\left(1 - \tfrac{1}{2}e^{-j\hat{\omega}_0}\right) = 7e^{j\pi/4}$$

$$\Rightarrow Ae^{j\varphi} = \frac{7e^{j\pi/4}}{1 - \frac{1}{2}e^{-j\pi/2}} = \frac{7e^{j\pi/4}}{1 + \frac{1}{2}j} = 6.26e^{j0.1\pi}$$

$\therefore$

$y[n] = 6.26\cos(\frac{\pi n}{2} + 0.1\pi)$