PROBLEM:

Suppose that a discrete-time signal $x[n]$ is given by the formula

$$x[n] = 333\cos(0.35\pi n - \pi/3)$$

and that it was obtained by sampling a continuous-time signal at a sampling rate of $f_s = 2500$ samples/second.

(a) Determine two *different* continuous-time signals $x_1(t)$ and $x_2(t)$ whose samples are equal to $x[n]$; i.e., find $x_1(t)$ and $x_2(t)$ such that $x[n] = x_1(nT) = x_2(nT)$ if $T = .0004$ sec. Both of these signals should have a frequency less than 2500 Hz. Give a formula for each signal.

(b) Determine the amplitude and phase for both of the signals found in part (a).

McClellan, Schafer and Yoder, *Signal Processing First*, ISBN 0-13-065562-7.
Prentice Hall, Upper Saddle River, NJ 07458.

SOLUTION

(a) $\hat{\omega}_0 = 0.35\pi$, $f_s = 2500\text{ Hz}$ $\Rightarrow 2\pi \dfrac{f_0}{f_s} = 0.35\pi \Rightarrow f_0 = \dfrac{.35}{2} f_s$

$$\Rightarrow f_0 = 437.5\text{ Hz}$$

A second possibility is the "folded" alias at $f_s - f_0 = 2500 - 437.5 = 2062.5\text{ Hz}$

(b) Let $x(t) = A\cos(2\pi f_0 t + \varphi)$

Then $x[n] = x(t)\big|_{t=\frac{n}{f_s}} = A\cos\left(2\pi \frac{f_0}{f_s} n + \varphi\right)$.

$\Rightarrow$ $A = 333$, $\varphi = -\pi/3$ when $f_0 = 437.5\text{ Hz}$

-OR- "Folded" case:

Let $x(t) = A\cos(2\pi(f_s - f_0)t + \varphi)$

$$x[n] = x\left(\frac{n}{f_s}\right) = A\cos\left(2\pi \frac{(f_s - f_0)}{f_s} n + \varphi\right)$$

$$= A\cos\left(2\pi n - 2\pi \frac{f_0}{f_s} n + \varphi\right)$$

$$= A\cos\left(2\pi \frac{f_0}{f_s} n - \varphi\right)$$

SIGN CHANGE ON PHASE

$\Rightarrow$ $A = 333$, $\varphi = +\pi/3$ when $\text{freq} = 2062.5\text{ Hz}$

McClellan, Schafer, and Yoder, *Signal Processing First*, ISBN 0-13-065562-7.
Prentice Hall, Upper Saddle River, NJ 07458. © 2003 Pearson Education, Inc.