

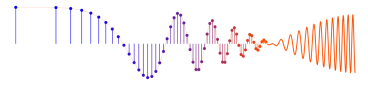
## PROBLEM:

Suppose that a discrete-time signal  $x[n]$  is given by the formula

$$x[n] = 333 \cos(0.35\pi n - \pi/3)$$

and that it was obtained by sampling a continuous-time signal at a sampling rate of  $f_s = 2500$  samples/second.

- Determine two *different* continuous-time signals  $x_1(t)$  and  $x_2(t)$  whose samples are equal to  $x[n]$ ; i.e., find  $x_1(t)$  and  $x_2(t)$  such that  $x[n] = x_1(nT) = x_2(nT)$  if  $T = .0004$  sec. Both of these signals should have a frequency less than 2500 Hz. Give a formula for each signal.
- Determine the amplitude and phase for both of the signals found in part (a).



$$(a) \left. \begin{array}{l} \hat{\omega}_0 = 0.35\pi \\ f_s = 2500 \text{ Hz} \end{array} \right\} \Rightarrow 2\pi \frac{f_0}{f_s} = 0.35\pi \Rightarrow f_0 = \frac{.35}{2} f_s \\ \Rightarrow f_0 = 437.5 \text{ Hz}$$

A second possibility is the "folded" alias at  $f_s - f_0 = 2500 - 437.5 = 2062.5 \text{ Hz}$

$$(b) \text{ Let } x(t) = A \cos(2\pi f_0 t + \varphi)$$

$$\text{Then } x[n] = x(t) \Big|_{t=\frac{n}{f_s}} = A \cos\left(2\pi \frac{f_0}{f_s} n + \varphi\right).$$

$$\Rightarrow A = 333 \quad \text{when } f_0 = 437.5 \text{ Hz} \\ \varphi = -\pi/3$$

-OR- "Folded" case:

$$\text{Let } x(t) = A \cos(2\pi (f_s - f_0) t + \varphi)$$

$$x[n] = x\left(\frac{n}{f_s}\right) = A \cos\left(2\pi \frac{(f_s - f_0)}{f_s} n + \varphi\right)$$

$$= A \cos\left(2\pi n - 2\pi \frac{f_0}{f_s} n + \varphi\right)$$

$$= A \cos\left(2\pi \frac{f_0}{f_s} n - \varphi\right)$$

SIGN CHANGE  
ON PHASE

$$\Rightarrow A = 333 \quad \text{when } \text{freq} = 2062.5 \text{ Hz} \\ \varphi = +\pi/3$$