PROBLEM:

Answer the following questions about the system whose z-transform system function is

$$H(z) = \frac{1 + z^{-2}}{1 + 0.77z^{-1}}$$

(a) Determine the poles and zeros of $H(z)$.

(b) Determine the difference equation relating the input and output of this filter

(c) Derive a simple expression (purely real) for the mag-squared of the frequency response $|H(e^{j\hat{\omega}})|^2$.

(d) Is this filter a Lowpass or Highpass filter? EXPLAIN your answer.

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SOLUTION

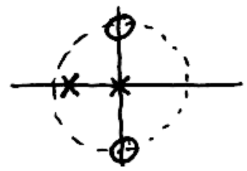
Answer the following questions about the system whose z-transform system function is

$$H(z) = \frac{1+z^{-2}}{1+0.77z^{-1}} = \frac{z^2+1}{z(z+0.77)}$$

(a) Determine the poles and zeros of $H(z)$.

ZEROS: roots of $z^2+1 \longrightarrow \{+j, -j\}$

POLES: roots of $z(z+0.77) \longrightarrow \{0, -0.77\}$



(b) Determine the difference equation relating the input and output of this filter

$$y[n] = -0.77\,y[n-1] + \underbrace{x[n] + x[n-2]}_{\{b_k\} \text{ for FIR part } = \{1,0,1\}}$$

(c) Derive a simple expression (purely real) for the mag-squared of the frequency response $|H(e^{j\hat{\omega}})|^2$.

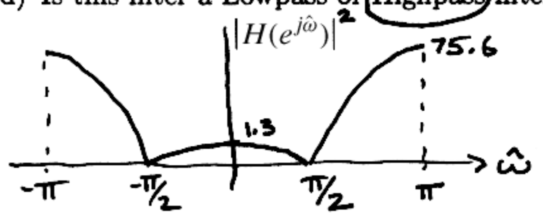
$$H(e^{j\hat{\omega}}) = H(z)\Big|_{z=e^{j\hat{\omega}}} = \frac{1+e^{-j2\hat{\omega}}}{1+0.77e^{-j\hat{\omega}}}$$

$$|H(e^{j\hat{\omega}})|^2 = H(e^{j\hat{\omega}})\,H^*(e^{j\hat{\omega}}) = \frac{1+e^{-j2\hat{\omega}}}{1+0.77e^{-j\hat{\omega}}} \cdot \frac{1+e^{+j2\hat{\omega}}}{1+0.77e^{+j\hat{\omega}}}$$

$$= \frac{1+1+e^{j2\hat{\omega}}+e^{-j2\hat{\omega}}}{1+.5929+0.77e^{j\hat{\omega}}+0.77e^{-j\hat{\omega}}}$$

$$= \frac{2+2\cos 2\hat{\omega}}{1.5929+1.54\cos\hat{\omega}} = \begin{cases} 4/3.13 = 1.28 & \text{at } \hat{\omega}=0 \\ 0 & \text{at } \hat{\omega}=\pi/2 \\ 4/.053 = 75.6 & \text{at } \hat{\omega}=\pi \end{cases}$$

(d) Is this filter a Lowpass or Highpass filter? EXPLAIN your answer.



From pole-zero plot above, pole is near $z=-1 \Rightarrow$ HPF

Also, a few evaluations of $H(e^{j\hat{\omega}})$ confirm the "high-pass" nature.

It is HIGH PASS because there is high gain at $\hat{\omega}=\pi$

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