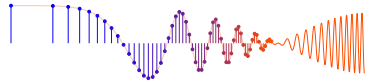


PROBLEM:

A signal composed of sinusoids is given by the following MATLAB code:

```
dt = 1/250;
tt = 0 : dt : 0.5;
xx = 9*cos(400*pi*tt+pi/3) + 8*sin(200*pi*tt) - 5 cos(1000*pi*tt);
```

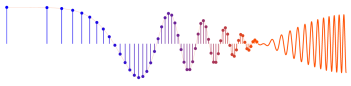
(a) For the signal vector `xx`, determine the correct formula for the discrete-time signal in the form: $x[n] = \sum_{k=1}^{N} A_k \cos(\hat{\omega}_k n + \phi_k)$

(b) Convert all the digital frequencies ($\hat{\omega}_k$) in part (a) to the range $-\pi < \hat{\omega} \leq \pi$.

(c) Sketch the "digital" spectrum of this signal indicating the complex phasor value at each frequency. Only the range $-\pi < \hat{\omega} \leq \pi$ needs to be shown.

McClellan, Schafer and Yoder, *Signal Processing First*, ISBN 0-13-065562-7.
Prentice Hall, Upper Saddle River, NJ 07458.

SOLUTION



The MATLAB code is equivalent to an A/D:
Sample at $F_s = 250$ Hz.

(a) $$x(t) = 9\cos(400\pi t + \pi/3) + 8\sin(200\pi t) - 5\cos(1000\pi t)$$

$$x[n] = x(t)\Big|_{t=n/F_s}$$

$$x[n] = 9\cos\left(\frac{400\pi n}{250} + \pi/3\right) + 8\cos\left(\frac{200\pi n}{250} - \pi/2\right) - 5\cos\left(\frac{1000\pi n}{250}\right)$$

$$= 9\cos(1.6\pi n + \pi/3) + 8\cos(0.8\pi n - \pi/2) - 5\cos(4\pi n)$$

subtract $2\pi n$ — This is FOLDING

subtract $4\pi n$ — $\cos(0) = 1$

$$\cos(-0.4\pi n + \pi/3) \longrightarrow \cos(0.4\pi n - \pi/3)$$

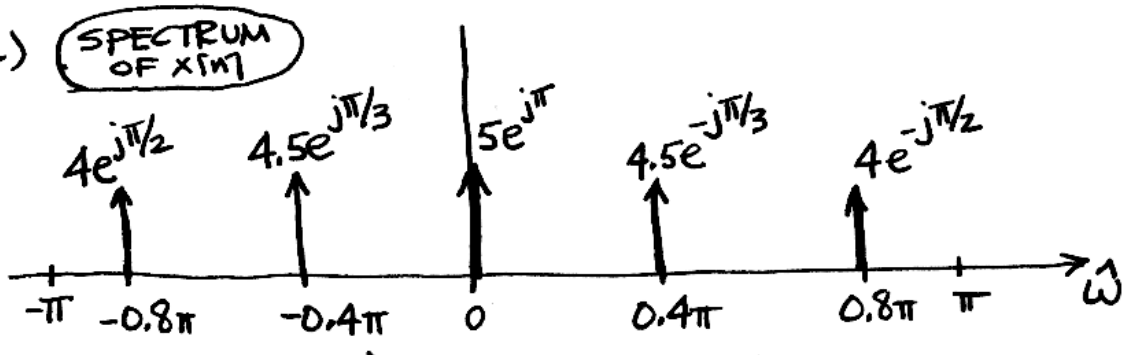
(b) $$x[n] = 9\cos(0.4\pi n - \pi/3) + 8\cos(0.8\pi n - \pi/2) - 5$$

$A_1 = 9$, $\varphi_1 = -\pi/3$, $\hat{\omega}_1 = 0.4\pi$

$A_2 = 8$, $\varphi_2 = -\pi/2$, $\hat{\omega}_2 = 0.8\pi$

$A_3 = 5$, $\varphi_3 = \pi$, $\hat{\omega}_3 = 0$

(c) SPECTRUM OF $x[n]$



For example,

$$9\cos(0.4\pi n - \pi/3) = \frac{9}{2}e^{-j(0.4\pi n - \pi/3)} + \frac{9}{2}e^{+j(0.4\pi n - \pi/3)}$$