PROBLEM:

A *unit impulse sequence* is defined as

$$\delta[n] = \begin{cases} 1 & n = 0 \\ 0 & n \neq 0 \end{cases}$$

Suppose that a LTI system has a z-transform system function equal to

$$\mathcal{H}(z) = 1 - z^{-1} - z^{-3} + z^{-4}$$

(a) Determine the difference equation that relates the output $y[n]$ of the system to the input $x[n]$.

(b) Determine and plot the *impulse response*: i.e., the output sequence $y[n]$ when the input is $x[n] = \delta[n]$. How is the output due to an impulse related to $\mathcal{H}(z)$?

(c) Determine the output of the system when the input is a shifted and scaled impulse:

$$x[n] = 7\delta[n-3]$$

(d) Determine the step response, i.e., the output when the input is

$$u[n] = \begin{cases} 1 & \text{for } n \geq 0 \\ 0 & n < 0 \end{cases}$$

McClellan, Schafer and Yoder, *Signal Processing First*, ISBN 0-13-065562-7.
Prentice Hall, Upper Saddle River, NJ 07458.

SOLUTION

(a) $H(z) = \sum_{k=0}^{M} b_k z^{-k} = 1 - z^{-1} - z^{-3} + z^{-4}$

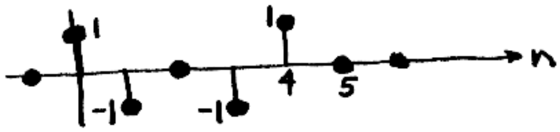
$\Rightarrow$ Filter coeffs are: $b_0 = 1$, $b_1 = -1$, $b_2 = 0$, $b_3 = -1$, $b_4 = 1$

Difference Eqn:

$$y[n] = x[n] - x[n-1] - x[n-3] + x[n-4]$$

(b) Impulse Response will "read out" the filter coeffs.

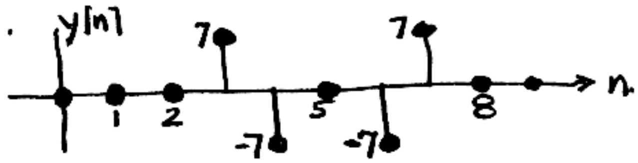
$$h[n] = \delta[n] - \delta[n-1] - \delta[n-3] + \delta[n-4]$$



(c) Use Linearity & Time Invariance Properties.

$x[n] = 7\delta[n-3] \Rightarrow$ OUTPUT IS $7h[n]$, but then shifted by 3.

$y[n] = 7h[n-3]$.



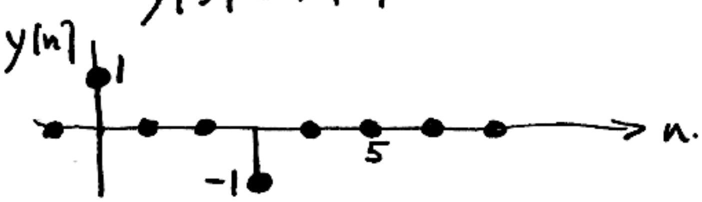
(d) INPUT $= u[n]$.

MAKE A TABLE

n	<0	0	1	2	3	4	5	6	....
x[n]	0	1	1	1	1	1	1	1	....
y[n]	0	1	0	0	-1	0	0	0	....

$y[2] = x[2] - x[1] - x[-1] + x[-2] = 1 - 1 - 0 + 0 = 0$

$y[3] = x[3] - x[2] - x[0] + x[-1] = 1 - 1 - 1 + 0 = -1$



McClellan, Schafer, and Yoder, *Signal Processing First*, ISBN 0-13-065562-7.